SINR Analysis of an IRS-Assisted MU-MISO System

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2026 International Conference on Signal Processing and Communications (SPCOM) | 979-8-3195-2494-2/26/$31.00 ©2026 IEEE | DOI: 10.1109/SPCOM71105.2026.11622945

***Abstract*—In this work, we characterize the outage probability (OP) of an intelligent reflecting surface (IRS) assisted multi-user multiple-input-single-output (MU-MISO) communication system. Using a two-step approximation method, we approximate the signal-to-interference-plus-noise ratio (SINR) for any downlink user by a Log-Normal random variable. The impact of various system parameters is studied using the closed-form expression of OP. It is concluded that the position of IRS has a critical role, but an appropriate increase in the number of IRS elements would help to compensate for the loss in performance if the position of IRS is suboptimal.**

***Index Terms*—Intelligent Reflecting Surface, Outage Probability, multi-user communications.**

I. Introduction

Intelligent reflecting surface (IRS) has gained much interest in the wireless communication research community thanks to its promise to enhance the performance of wireless communication systems. IRS is envisioned to be one of the critical enablers of technology for future generations of wireless communication [1]–[3]. IRS is a hypersurface made up of metamaterials that can interact with impinging electromagnetic (EM) wave and configure it in a programmed manner. A hypersurface planner array consists of several independent meta-atoms [4]. These meta-atoms, also known as IRS elements, can reflect the incident EM wave by configuring the amplitude and phase of the wave [5].

Outage probability (OP) is an important metric to characterize the performance of a communication system. Works like [6]–[8] have focused on the characterization of OP for an IRS-assisted single-input-single-output (SISO) systems. IRS-assisted beam index modulation schemes for improved spectral efficiency were proposed in [9]. A closed form expression for optimal BF vector is proposed in [10] for a single-input-multiple-output system model. Performance analysis of an IRS-assisted multiple-input-single-output (MISO) communication system under a deterministic source to IRS link has been done in [11]. The authors of [12] considered a single-user MISO system and derived the bit error probability by approximating the effective channel between source and user as a Gamma random variable (RV). In [13], the stochastic gradient descent method was utilized to jointly design the phase shift at IRS and beamforming (BF) vector at source to minimize the OP where they used the empirical definition of OP as the objective function. The authors of [14] consider a multi-user MISO (MU-MISO) system and provide a lower bound on the number of IRS elements such that the sum rate is greater than a threshold asymptotically in the absence of line-of-sight (LoS) link. Similarly, the authors in [15] analyze the asymptotic max-min the signal-to-interference-plus-noise ratio (SINR) in IRS-assisted MU-MISO systems in the absence of LoS. Recently, authors of [16] studied the effect of hardware impairments on an IRS-assisted single-user MISO system where the LoS is assumed to be blocked. They approximated the amplitude of the effective cascaded channel between source and user as Gamma RV using the method of moments. Extending this, the authors in [17] analyze SINR distribution for secrecy analysis.

Motivated by this research gap, in this work, we considered an IRS-assisted MU-MISO system. In a multi-user scenario, SINR is a ratio of two correlated random variables, which makes the statistical characterization of SINR a non-trivial problem. We propose a simple expression for the probability density function (PDF)/ cumulative distribution function (CDF) of SINR using a two-step approximation. The approach developed in this work is quite general and can be easily applied to analyze the OP for a system with other general fading channels.

Notation: In this work, $\mathcal{LN}(\mu, \sigma^2)$ represent the Log-Normal distribution with parameters μ and σ, $\mathrm{diag}(\theta_1, \cdots, \theta_N)$ denotes a diagonal matrix with entries $\theta_1, \cdots, \theta_N$ and $\arg(\mathrm{z})$ denotes the argument (phase) of the complex number z. $\mathrm{Cov}(\cdot)$ is the covariance, $\ln$ is the natural log and $\mathrm{erfc}(\cdot)$ is the complementary error function [18]

II. System Model

We consider an IRS-assisted downlink MISO system similar to the model studied in [14]. The source, denoted by $\mathbf{S}$, is equipped with M antennas that serve K users, each equipped with a single antenna. The number of IRS elements is assumed to be N. The distances from $\mathbf{S}$ to IRS and IRS to k-th user (U_k) are represented by d_{sr} and d_{rk}, respectively. We assume that the dimensions of IRS elements are small compared to the distances d_{rk} and d_{sr}; hence, all the IRS elements experience the same fast fading. The presence of a blockage between $\mathbf{S}$ and U_k is assumed such that there is no direct link between $\mathbf{S}$ and U_k.

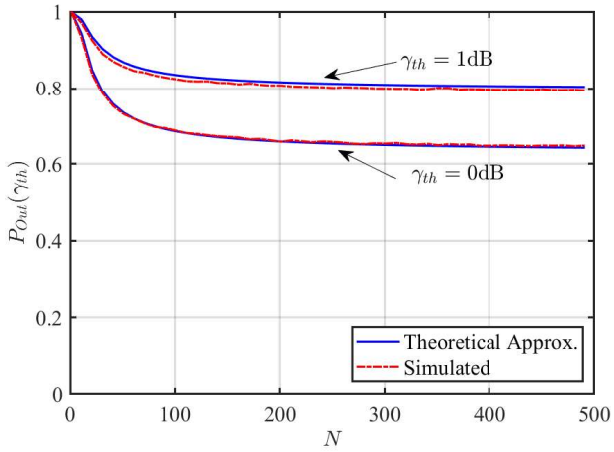


Fig. 5: Effect of N on OP

V. Conclusion

In this work, we have approximated the SINR of an IRS-assisted MU-MISO system by a Log-Normal RV. Based on this, we have characterized the OP in an MU-MISO system without a direct LoS link. Numerical results confirmed the closeness of the proposed approximation to the simulated OP. We also studied the impact of IRS position and the number of IRS elements on the OP. It is shown that a fundamental limit exists up to which IRS improves performance. The tractable closed-form approximation provided here can also be used to derive other performance metrics, which are functions of the SINR. Also, the methodology presented in this work is general and can be extended to other fading channel scenarios.

Acknowledgment

The authors thank Athira Subash for her valuable contributions and collaboration during the early stages of this work.

Appendix A
Proof of Lemma 1

From (2), we have $X_k = \lambda_k^2(\mathbf{g}_k\mathbf{g}_k^H)^2$. Let, $\mathbb{E}\left[|h_{rk}^j|^i\right] = m_i^{rk}$, $\mathbb{E}\left[|\mathbf{g}_{sr}^j(l)|^i\right] = m_i^{sr}$, $\forall i = \{2,4,6,8\}$, $m_2^{xy} = \alpha_{xy} = d_{xy}^{-\beta}$ where $xy \in \{sr, rk, rj, rh\}$. From the property of moments of Rayleigh distribution, $m_4^{xy} = 2\alpha_{xy}^2$, $m_6^{xy} = 6\alpha_{xy}^3$, $m_8^{xy} = 24\alpha_{xy}^4$, where $xy \in \{sr, rk, rj, rh\}$. After expanding and substituting, the first moment is obtained as,

$$\mathbb{E}[X_k] = \lambda_k^2\alpha_{rk}^2\alpha_{sr}^2 MN(M+1)(N+1). \tag{8}$$

Similarly, the second moment is evaluated as

$$\begin{aligned}\mathbb{E}\left[X_k^2\right] = \lambda_k^4\alpha_{rk}^4\alpha_{sr}^4 MN(48\alpha_{sr}^2(M-2)(M-1)(N-1)\\ +M^3(N+1)(N+2)(N+3)+6M^2(N(N(N+6)+3)\\ +14)+M(N(11N(N+6)+265)-78)+6(N(N(N+6)\\ -5)+22)).\end{aligned} \tag{9}$$

Appendix B
Proof of Lemma 2

From (2), we have $Y_k = \underbrace{\sum_{j\neq k}^{K}\lambda_j^2(\mathbf{g}_k\mathbf{g}_j^H\mathbf{g}_j\mathbf{g}_k^H)}_{Z_k} + \sigma_k^2$. Taking the term-by-term expectation, we have

$$\begin{aligned}\mathbb{E}[Y_k] &= \mathbb{E}[Z_k] + \sigma_k^2 \\ &= MN(M+N)\alpha_{rk}\alpha_{sr}^2\left(\sum_{j\neq k}^{K}\lambda_j^2\alpha_{rj}\right) + \sigma_k^2.\end{aligned} \tag{10}$$

The second moment of Y_k is

$$\mathbb{E}\left[Y_k^2\right] = \mathbb{E}\left[Z_k^2\right] + 2\mathbb{E}[Z_k]\sigma_k^2 + \sigma_k^4 \tag{11}$$

The second moment of Z_k is obtained as,

$$\begin{aligned}\mathbb{E}\left[Z_k^2\right] = \sum_{j\neq k}^{K}\lambda_j^4\mathbb{E}\left[\underbrace{\left(\mathbf{g}_k\mathbf{g}_j^H\mathbf{g}_j\mathbf{g}_k^H\right)^2}_{T_1}\right] \\ +\sum_{j\neq k}^{K}\sum_{h\neq k;h\neq j}^{K}\lambda_j^2\lambda_h^2\mathbb{E}\left[\underbrace{\mathbf{g}_k\mathbf{g}_j^H\mathbf{g}_j\mathbf{g}_k^H\mathbf{g}_k\mathbf{g}_h^H\mathbf{g}_h\mathbf{g}_k^H}_{T_2}\right]\end{aligned} \tag{12}$$

Expanding each sum and taking the moment of the T_1, we get

$$\begin{aligned}\mathbb{E}[T_1] = 2MN(M+1)(N+1)(M+N+1)\\ \times(M+N+2)\alpha_{rj}^2\alpha_{rk}^2\alpha_{sr}^4\end{aligned} \tag{13}$$

Similarly, for T_2, the moment obtained is

$$\begin{aligned}\mathbb{E}[T_2] = MN(M+1)(N+1)(M+N+1)\\ \times(M+N+2)\alpha_{rh}\alpha_{rj}\alpha_{rk}^2\alpha_{sr}^4\end{aligned} \tag{14}$$

After substituting values from (13) and (14) in (12), we have

$$\begin{aligned}\mathbb{E}\left[Z_k^2\right] = A_{M,N}\alpha_{rk}^2\alpha_{sr}^4\left(\sum_{j\neq k}^{K}\lambda_j^4\alpha_{rj}^2\right)\\ +B_{M,N}\left(\mathbb{E}[Z_k]\right)^2\end{aligned} \tag{15}$$

where $A_{M,N} = MN(M+1)(N+1)(M+N+1)(M+N+2)$ and $B_{M,N} = \frac{(M+1)(M+N+1)(M+N+2)}{MN(N+1)}$.

Appendix C
Expression for $\mathrm{Cov}(\ln(X_k), \ln(Y_k))$

The covariance between $\ln(X_k)$ and $\ln(Y_k)$ under the Log-Normal assumption is [29]

$$\mathrm{Cov}(\ln(X_k), \ln(Y_k)) = \ln\left(\frac{\mathrm{Cov}(X_k, Y_k)}{\mathbb{E}[X_k]\mathbb{E}[Y_k]} + 1\right) \tag{16}$$

The covariance between X_k, Y_k is defined as $\mathrm{Cov}(X_k, Y_k) = \mathrm{Cov}(X_k, Z_k) = \mathbb{E}[(X_k - \mathbb{E}[X_k])(Y_k - \mathbb{E}[Y_k])]$

$$\mathrm{Cov}(X_k, Y_k) = \mathbb{E}[X_k Z_k] - \mathbb{E}[X_k]\mathbb{E}[Z_k], \tag{17}$$

where $\mathbb{E}[X_k Z_k]$ is obtained as follows,

$$\mathbb{E}[X_k Z_k] = \alpha_{rk}^3\alpha_{sr}^4\lambda_k^2 C_{M,N}\sum_{j\neq k}^{K}\lambda_j^2\alpha_{rk}, \tag{18}$$

where $C_{M,N} = MN(M^3(1+N)(2+N)+M^2(1+N)(2+N)(5+N)+M(1+N)(2+N)(8+3N)+2(-1+N)(14+N(6+N)))$

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