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Modified Boltzmann sigmoidal model for predicting functional fatigue behaviour of NiTi shape memory alloys

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Abstract

Shape memory alloys (SMAs), particularly NiTi-based alloys, exhibit functional fatigue under cyclic thermomechanical loading, characterized by permanent strain accumulation, recovery strain reduction, and hysteresis shifts, which degrade long-term performance. This study presents a simplified and accurate model based on a modified Boltzmann sigmoidal (MBS) function, where the original parameters were systematically redefined to represent key SMA properties, such as recovery strain, permanent strain, transformation temperature, and cycle number. The model was developed and validated using experimental data obtained from NiTi SMA samples subjected to 500 cycles at constant stress levels (50 MPa and 60 MPa), with strain measurements captured using a high-resolution LASER displacement sensor. The MBS model effectively predicted the exponential evolution of recovery and permanent strains with increasing cycles and achieved a high curve-fitting accuracy ($R^2 > 0.98$). Compared to complex models like the Limiting Loop Proximity (L^2P) model, the proposed approach is computationally efficient, easy to implement, and requires minimal calibration, making it well-suited for practical engineering applications involving SMA-based components operating under cyclic conditions.

Keywords NiTi SMAs · Functional fatigue · Recovery strain · Permanent strain · L^2P model · Modified Boltzmann sigmoidal (MBS) model

1 Introduction

Shape memory alloys (SMAs) are a class of smart materials that exhibit the ability to recover their original shape upon thermal or thermomechanical activation. This unique property arises due to a reversible, diffusionless martensitic phase transformation between a high-temperature austenitic phase and a low-temperature martensitic phase. NiTi-based SMAs, in particular, have gained widespread attention due to their superior mechanical properties, including high ductility, excellent corrosion resistance, good wear resistance, significant recoverable strain, and exceptional energy absorption capacity [1]. These properties make NiTi SMAs ideal for applications in biomedical, aerospace, and automotive industries, where cyclic loading is a critical consideration [2].

Cyclic loading plays a crucial role in many SMA applications, including actuators, medical stents, and seismic dampers. Under repeated loading, SMAs exhibit two major types of degradation: structural fatigue and functional fatigue [3]. Structural fatigue refers to mechanical failure due to microstructural damage accumulation, whereas functional fatigue affects the essential properties of the material, such as recovery strain (amount of strain recovered during the reverse transformation from martensite to austenite), transformation temperatures (critical temperatures at which forward and reverse phase transformations occur), and hysteresis width (temperature difference between the forward and reverse phase transformations). The progressive accumulation or permanent strain (residual deformation that remains in the SMA after unloading) of functional fatigue can significantly reduce the performance and reliability of

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SMA components over time, limiting their practical applications in cyclic environments (Lagoudas et al., 2009).

Functional fatigue in NiTi SMAs is primarily driven by dislocation accumulation, microstructural evolution, and phase transition dynamics during cyclic phase transformation. The number of cycles required to reach a steady functional state is influenced by several factors, including dislocation density, grain size, chemical composition, thermomechanical processing history, and operational conditions [4]. Studies have shown that prolonged cyclic loading results in work hardening due to interactions between dislocations, which eventually leads to a plateau in functional degradation, establishing a stable behaviour. However, the exact mechanisms governing functional fatigue remain complex and require further investigation [5–7].

Several modelling approaches have been proposed to describe SMA behaviour under cyclic loading. Constitutive models, such as the Lagoudas thermomechanical model, have been widely used to capture the stress-strain response and phase transformation kinetics of SMAs [8]. Additionally, dislocation-based models have been developed to predict microstructural evolution and its impact on functional properties [9]. A phase-field model for fracture and fatigue in SMAs captures superelasticity, phase transformation, and damage evolution, accurately predicting crack propagation, stress-induced toughening, and fatigue behaviour in SMA components like biomedical stents [10].

However, these models often involve complex mathematical formulations and computationally expensive simulations, making them less practical for real-time applications. Another class of models includes phenomenological approaches that use empirical data to describe SMA behaviour. The Limiting Loop Proximity model, originally developed for magnetic hysteresis, has been adapted to describe SMA thermomechanical hysteresis [11]. Although this model provides an algebraic representation of hysteresis, it is computationally intensive and does not effectively capture the accumulation of functional fatigue over multiple cycles.

As SMAs are used in advanced applications, such as actuators and biomedical devices, their performance often decreases over time due to functional fatigue caused by repeated loading. Predicting this degradation is essential for improving the durability and reliability of SMA components. Therefore, there is a need for a simple, accurate, and fast model that can predict this behaviour efficiently.

To address this gap, the present study proposes a Modified Boltzmann Sigmoidal (MBS) model that incorporates key functional fatigue parameters, such as recovery strain, permanent strain, and cyclic hysteresis changes. Sigmoidal functions like Gaussian cumulative, Lorentz cumulative, and Boltzmann sigmoid are commonly used to describe phase transformations [12, 13]. Among these, the Boltzmann function is selected due to its good fit with strain-temperature behaviour and its ability to represent SMA characteristics geometrically.

In this work, the Boltzmann function is modified to account for the changes in SMA behaviour over multiple thermomechanical cycles. The proposed MBS model is tested and validated using experimental data from NiTi SMA samples subjected to 500 cycles at constant stress. The model shows high accuracy in predicting strain evolution and is computationally efficient. This approach provides a practical tool for evaluating long-term SMA performance and supports the design of reliable components for applications involving repeated loading.

2 Methodology

2.1 Modelling of functional fatigue behaviour of NiTi SMA

In this work, an attempt has been made to identify and develop a simple yet effective mathematical model to predict the functional fatigue behaviour of a NiTi SMA using two methods, namely, Limiting Loop Proximity (L^2P) model and modified Boltzmann sigmoidal (MBS) model.

2.1.1 Limiting loop proximity (L^2P) model

The Limiting Loop Proximity model, originally developed for magnetic hysteresis [11], has been adapted in previous studies to describe the thermomechanical hysteresis of SMAs. The model defines hysteresis behaviour in terms of algebraic equations that relate transformation temperatures and strain evolution under cyclic loading. The L^2P model defines the thermo-mechanical hysteresis using the following stroke-temperature equation. Initially, using the existing experimental data, the values of H_h, H_w, T, T_c and the curve-fitting constant was incorporated in Eq. 1.

$$s(T) = \frac{H_h}{\pi}\left[arctan\left(\beta\left(\delta\frac{H_w}{2} + T_c - T\right)\right) + \frac{\pi}{2}\right] \quad (1)$$

where,

H_h = Hysteresis height

H_w = Hysteresis width

T_c = Critical temperature, i.e. the temperature at the centre of the hysteresis curve

T = Temperature of the sample

β and δ = Curve-fitting constants for strain accumulation over cycles [9]. These models are accurate but need detailed material parameters and are difficult to implement. Phase-field models simulate the evolution of phase boundaries and local transformations during cycling, but they also involve extensive computation and fine mesh resolution [10]. In comparison, the proposed Modified Boltzmann Sigmoidal (MBS) model is simpler and easier to apply. It uses experimentally obtained strain–temperature data and curve fitting to describe the changes in recovery strain, permanent strain, and hysteresis with increasing cycles. Moreover, the proposed model achieves high accuracy ($R^2 > 0.98$) and does not require complex calibration or numerical simulations. This makes the MBS model suitable for quick estimation of functional fatigue in SMA components used in cyclic applications.

4 Conclusions

In the present study, an attempt was made to develop a model to address the functional fatigue behaviour of NiTi SMAs. Functional fatigue experiments were conducted at a stress level of 50 MPa and a maximum temperature of 65° C for 500 cycles. The existing model, i.e., the L^2P model, was used to understand the transformation behaviour of the SMAs. However, it seems to be more complex in nature and involves many unknown variables, which has led to its impracticability. The Boltzmann-sigmoidal model was adopted and modified so as to account for the shape memory behaviour of SMAs. The aforementioned model was then further modified to account for the functional fatigue behaviour of SMAs by including the parameters which accounted for the number of cycles. A new set of experiments was conducted to validate the modified Boltzmann sigmoidal (MBS) model, and the accuracy of the curve fitted was measured. It was found that the established MBS model fits with the experimental values with an accuracy of 98%. The proposed model addresses a key challenge in SMA by predicting functional fatigue with high accuracy and low complexity, making it a practical tool for engineers designing SMA components for cyclic applications.

Author contributions *G. Swaminathan*: Conceptualization, Methodology, Writing– Original Draft *S.H. Adarsh*: Data Curation, Validation *M. Bharathi*: Investigation, Writing– Review & Editing.

Data availability The raw/processed data analysed in the current study are available from the corresponding author upon reasonable request.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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