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ORIGINAL PAPER

Performance analysis of an M/G/1 retrial queue with general retrial time, modified M-vacations and collision

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Abstract In this paper, a single server retrial queue with general retrial time and collisions of customers with modified M-vacations is studied. The primary calls arrive according to Poisson process with rate λ. If the server is free, the arriving customer/the customer from orbit gets served completely and leaves the system. If the server is busy, arriving customer collides with the customer in service resulting in both being shifted to the orbit. After the collision the server becomes idle. If the orbit is empty the server takes at most M vacations until at least one customer is recorded in the orbit when the server returns from a vacation. Whenever the orbit is empty the server leaves for a vacation of random length V. If no customers appear in the orbit when the server returns from vacation he again leaves for another vacation with the same length. This pattern continues until he returns from a vacation to find at least one customer recorded in the orbit or he has already taken M vacations. If the orbit is empty by the end of the Mth vacation, the server remains idle for customers in the system. The time between two successive retrials from the orbit is assumed to be general with arbitrary distribution R(t). By applying the supplementary variables method, the probability generating function of number of customers in the orbit is derived. Some special cases are also discussed. A numerical illustration is also presented.

Keywords General retrial time · Collision · Modified M-vacations · Retrial queue · Supplementary variable method

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1 Introduction

In recent years there have been significant contributions to the retrial queueing system. A retrial queueing system is characterized by the features that the arriving calls, which find a server busy, do not line up or leave the system immediately forever, but they go to some virtual place called an orbit and try their luck after some random time. Such queueing systems play important roles in the analysis of many telephone switching systems, telecommunication networks, computer systems, local area networks and daily life situations. A review of retrial queue literature could found in Yang and Templeton (1987), Falin and Templeton (1997), Artalejo (1999a, b) and Artalejo and Gomez-coral (2008). A number of applications of retrial queues in science and engineering can be found in Kulkarni and Liang (1997). For many applications in telecommunications and mobile communication, Choi and Park (1990), Choi et al. (1995), Choi and Chang (1999) studied the single server retrial queue with priority calls and Krishnakumar et al. (2002) analysed an M/G/1 retrial queue with feedback and starting failures using supplementary variable technique.

Queueing systems with vacation time have been found to be useful in modeling the systems in which the server has additional tasks. Various authors have analyzed queueing problems of server vacations with several combinations. A literature survey on queueing systems with server vacation can be found in Doshi (1986). Doshi (1985) discussed an M/G/1/system with variable vacations. A comprehensive and excellent study on the vacation models can be found in Takagi (1991). Li and Yang (1995) developed an M/G/1 retrial system with server vacations. Later Artalejo (1997) analyzed an M/G/1 retrial queue with exhaustive server vacations, i.e. the server takes a vacation only when there are no customers in the systems. Krishna Reddy and Anitha (1999) considered an M/G(a, b)/1 model with M different types of vacations. Arumuganathan et al. (2008) gave an excellent study on steady state analysis of a non-Markovian bulk queueing system with N-policy and different types of vacations. Krishna kumar et al. (2002) studied an M/G/1 retrial queue where the server operate according to a Bernoulli vacation policy as described by Keilson and Servi (1986).

In many situations involving data transmission from diverse sources there can be conflict for a limited number of channels or other facilities. Uncoordinated attempts by several sources to use a single server facility can result in “Collision” leading to the loss of the transmission. Jonin (1982) and Falin and Sukharev (1985) have analyzed the retrial queueing system with collision, called the queue with double connections, in which, if an arriving customer interrupts (collides with) a customer in service, both the arriving customer and the served customer join the retrial group and the server becomes free immediately. Choi et al. (1992) have discussed a retrial queueing system with constant retrial rate and collision in the specific communication protocol CSMA-CD. Recently Krishna kumar et al. (2010) analyzed a single server feedback retrial queue with collisions. Wu et al. (2011) have analyzed a retrial queue with pre emptive resume and collisions. Gao and Yao (2013) have discussed a queueing system with randomized working vacations and at most J vacations.

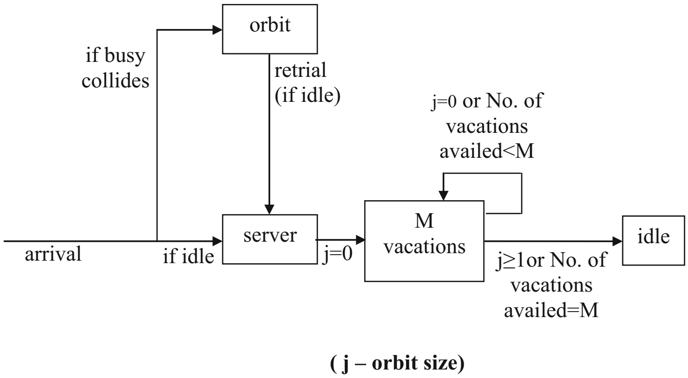

Fig. 1 Schematic representation of the queueing model

Several results have been reported separately on retrial queueing systems with general retrial time, retrial queues with modified vacations and retrial queues with collisions. The study of retrial queueing systems, taking into account the above mentioned features is worth investigating. Not much work in this direction is found in the literature. Based on this observation, a single server retrial queueing system has been discussed with general retrial time, modified M-vacations and collisions.

In this paper, we consider the case of an M/G/1 retrial queueing system with general retrial time, modified M-vacations and collision. At the arrival epoch if the server is idle, then the arriving customer begins its service immediately. Otherwise, at the arrival epoch if the server is busy, the arriving customer collides with the customer in service resulting in both being shifted to the orbit. After the collision, the server becomes idle. Whenever the orbit is empty the server leaves for a vacation. At a vacation completion epoch, if the orbit size is zero, the server leaves for another vacation of the same duration. This pattern continues until the server returns from a vacation to find at least one customer recorded in the orbit or until it has already availed M number of vacations. If the orbit is empty by the end of the Mth vacation, the server remains idle in the system to render service for customers from main pool or from retrial group. The model under study is schematically represented in Fig. 1.

2 Motivation

The motivation for the proposed model comes from a situation observed in the performance evaluation of local area networks operating under transmission protocols like the carrier sense multiple access with collision detection (CSMA-CD). For instance, in the CSMA-CD protocols for a fiber optic bus network with a finite number of stations, each of which has an infinite storage buffer, the collisions occur during the transmission of arbitrary length packets. This is because no slot synchronization is needed. Further, under the unslotted CSMA-CD protocol, transmission of deferred packets promptly begins the instant the channel is sensed to be idle. To ensure good functioning of the server, maintenance activities (i.e.

multiple vacations) such as virus scan can be performed when the server is idle. This type of maintenance can be programmed to perform on a regular basis. This situation can be modelled as an M/G/1 retrial queue with general retrial time, modified M-vacations and collision.

3 Mathematical model

The customers arrive according to a Poisson process with rate λ. The time between two successive retrials from the orbit is assumed to be general with arbitrary distribution R(t). Let $R(x)(r(x))\{\tilde{R}(\theta)\}[R^0(x)]$ be the cumulative distribution function (probability density function) {Laplace–Stieltjes transform} [remaining retrial time] of retrial time. Let $S(x)(s(x))\{\tilde{S}(x)\}[S^0(x)]$ be the cumulative distribution function (probability density function) {Laplace–Stieltjes transform}[remaining service time] of service. Let $V(x)(v(x))\{\tilde{V}(\theta)\}[V^0(x)]$ be the cumulative distribution function (probability density function) {Laplace–Stieltjes transform} [remaining vacation time] of vacation. N(t) denotes the number of customers in the orbit at time t.

The server state is denoted as

$$C(t) = \begin{cases} 0, & \text{if the server is idle} \\ 1, & \text{if the server is busy} \\ 2, & \text{if the server is on vacation} \end{cases}$$

Now the system state probabilities are defined as follows:

(1) $P_{00}(t) = P\{C(t) = 0; N(t) = 0\}; n \geq 0$ is the probability that at time t the server is idle and the orbit size is empty.
(2) $P_{0,n}(x,t)dt = P\{C(t) = 0; N(t) = n; x < R^0(t) \leq x + dt\}; n \geq 1$ is the probability that at time t the server is idle, the orbit size is n and the remaining retrial time of a customer at an arbitrary time is between x and x + dt.
(3) $P_{1,n}(x,t)dt = P\{C(t) = 1; N(t) = n; x < S^0(t) \leq x + dt\}; n \geq 1$ is the probability that at time t the server is busy, the orbit size is n and the remaining service time of a customer under service is between x and x + dt.
(4) $V_{l,n}(x,t)dt = P\{C(t) = 2; N(t) = n, x \leq V^0(t) \leq x + dt\}; l = 1, 2, \ldots M, n \geq 0$ is the probability that at time t the server is on the lth vacation, the orbit size is n and the remaining vacation time of a customer at an arbitrary time is between x and x + dt.

4 Steady state orbit size distribution

To derive the steady state orbit size distribution following equations are obtained using supplementary variable technique cox (Cox 1965),

$$P_{0,0}(t+\Delta t) = P_{0,0}(t)(1-\lambda\Delta t) + V_{M,0}(0,t)\Delta t$$

$$P_{0,j}(x-\Delta t, t+\Delta t) = P_{0,j}(x,t)(1-\lambda\Delta t) + P_{1,j}(0,t)r(x)\Delta t + \lambda(1-\delta_{i,j}) \times \left[\int_0^\infty P_{1,j-2}(x,t)dx\right] r(x)\Delta t + \sum_{l=1}^{M} V_{1,j}(0,t)\Delta t r(x), \quad j \geq 1$$

$$P_{1,0}(x-\Delta t, t+\Delta t) = P_{1,0}(x,t)(1-\lambda\Delta t) + \lambda\Delta t P_{0,0}(t)s(x) + P_{0,1}(0,t)s(x)\Delta t$$

$$P_{1,j}(x-\Delta t, t+\Delta t) = P_{1,j}(x,t)(1-\lambda\Delta t) + \lambda\Delta t)\left[\int_0^\infty P_{0,j}(x,t)dx\right] s(x) + P_{0,j+1}(0,t)s(x)\Delta t, \quad j \geq 1$$

$$V_{1,0}(x-\Delta t, t+\Delta t) = V_{1,0}(x,t)(1-\lambda\Delta t) + P_{1,0}(0,t)v(x)\Delta t$$

$$V_{1,j}(x-\Delta t, t+\Delta t) = V_{1,j}(x,t)(1-\lambda\Delta t) + \lambda\Delta t V_{1,j-1}(x,t), \quad j \geq 1$$

$$V_{l,0}(x-\Delta t, t+\Delta t) = V_{l,0}(x,t)(1-\lambda\Delta t) + V_{l-1,0}(0,t)v(x)\Delta t$$

$$V_{l,j}(x-\Delta t, t+\Delta t) = V_{1,j}(x,t)(1-\lambda\Delta t) + V_{1,j-1}(x,t)\lambda\Delta t, \quad j \geq 1, \quad 2 \leq l \leq M$$

where $\delta_{l,j} = \begin{cases} 0, & j \neq 1 \\ 1, & j = 1. \end{cases}$

From the above equations, the steady state queue size equations are obtained as follows:

$$\lambda P_{00} = V_{M,0}(0) \tag{1}$$

$$-\frac{d}{dx}P_{0,j}(x) = -\lambda P_{0,j}(x) + P_{1,j}(0)r(x) + \lambda(1-\delta_{1,j})\left[\int_0^\infty P_{1,j-2}(x)dx\right] r(x) + \sum_{l=1}^{M} V_{1,j}(0)r(x), \quad j \geq 1 \tag{2}$$

$$-\frac{d}{dx}P_{1,j}(x) = -\lambda P_{1,j}(x) + \lambda P_{00}s(x) + P_{0,1}(0)s(x) \tag{3}$$

$$-\frac{d}{dx}P_{1,j}(x) = -\lambda P_{1,j}(x) + \lambda\left[\int_0^\infty P_{0,j}(x)dx\right] s(x) + P_{0,j+1}(0)s(x), \quad j \geq 1 \tag{4}$$

$$-\frac{d}{dx}V_{1,0}(x) = -\lambda V_{1,0}(x) + P_{1,0}(0)v(x) \tag{5}$$

$$-\frac{d}{dx}V_{1,j}(x) = -\lambda V_{1,j}(x) + \lambda V_{1,j-1}(x), \quad j \geq 1 \tag{6}$$

$$-\frac{d}{dx}V_{l,0}(x) = -\lambda V_{l,0}(x) + V_{l-1,0}(0)v(x) \tag{7}$$

$$-\frac{d}{dx}V_{l,j}(x) = -\lambda V_{l,j}(x) + \lambda V_{l,j-1}(x), \quad j \geq 1, \quad 2 \leq l \leq M \tag{8}$$

The Laplace–Stieltjes transforms (LST) of $P_{1,j}(x)$, $V_{l,j}(x)$ are defined as

$$LST(P_{1,j}(x)) = \tilde{P}_{1,j}(\theta) = \int_0^\infty e^{-\theta x}P_{1,j}(x)dx;$$

$$LST(V_{l,j}(x)) = \tilde{V}_{l,j}(\theta) = \int_0^\infty e^{-\theta x}V_{l,j}(x)dx.$$

Taking Laplace–Stieltjes transform on steady state Eqs. (2)–(8) we have

$$\theta\tilde{P}_{0,j}(\theta) - P_{0,j}(0) = \lambda\tilde{P}_{0,j}(\theta) - P_{1,j}(0)\tilde{R}(\theta) - \sum_{l=1}^{M} V_{l,j}(0)\tilde{R}(\theta) - \lambda(1-\delta_{1,j})\left[\int_0^\infty P_{1,j-2}(x)dx\right]\tilde{R}(\theta), \quad j \geq 1 \tag{9}$$

$$\theta\tilde{P}_{1,0}(\theta) - P_{1,0}(0) = \lambda\tilde{P}_{1,0}(\theta) - \lambda P_{00}\tilde{S}(\theta) - P_{0,1}(0)\tilde{S}(\theta) \tag{10}$$

$$\theta\tilde{P}_{1,j}(\theta) - P_{1,j}(0) = \lambda\tilde{P}_{1,j}(\theta) - \lambda\left[\int_0^\infty P_{0,j}(x)dx\right]\tilde{S}(\theta) - P_{0,j+1}(0)\tilde{S}(\theta), \quad j \geq 1 \tag{11}$$

$$\theta\tilde{V}_{1,0}(\theta) - V_{1,0}(0) = \lambda\tilde{V}_{1,0}(\theta) - P_{1,0}(0)\tilde{V}(\theta) \tag{12}$$

$$\theta\tilde{V}_{1,j}(\theta) - V_{1,j}(0) = \lambda\tilde{V}_{1,j}(\theta) - \lambda\tilde{V}_{1,j}(\theta), \quad j \geq 1 \tag{13}$$

$$\theta\tilde{V}_{l,0}(\theta) - V_{l,0}(0) = \lambda\tilde{V}_{l,0}(\theta) - V_{l-1,0}(0)\tilde{V}(\theta) \quad 2 \leq l \leq M \tag{14}$$

$$\theta\tilde{V}_{l,j}(\theta) - V_{l,j}(0) = \lambda\tilde{V}_{l,j}(\theta) - \lambda\tilde{V}_{l,j-1}(\theta), \quad j \geq 1 \tag{15}$$

4.1 Probability generating function

To find the probability generating function (PGF) of the number of customers in the orbit at an arbitrary time epoch, the following PGFs are defined.

$$\begin{aligned}\tilde{P}_0(z,\theta) &= \sum_{j=1}^{\infty}\tilde{P}_{0,j}(\theta)z^j; \quad P_0(z,0) = \sum_{j=1}^{\infty}P_{0,j}(0)z^j;\\ \tilde{P}_1(z,\theta) &= \sum_{j=0}^{\infty}\tilde{P}_{1,j}(\theta)z^j; \quad P_1(z,0) = \sum_{j=0}^{\infty}P_{1,j}(0)z^j;\\ \tilde{V}_l(z,\theta) &= \sum_{j=0}^{\infty}\tilde{V}_{l,j}(\theta)z^j; \quad V_l(z,0) = \sum_{j=0}^{\infty}V_{l,j}(0)z^j.\end{aligned} \tag{16}$$

The PGF P(z) of number of customers in orbit at an arbitrary time instant can be expressed as follows

$$P(z) = P_{00} + \tilde{P}_0(z,0) + \tilde{P}_1(z,0) + \sum_{l=1}^{M} V_l(z,0) \tag{17}$$

In order to find $\tilde{P}_0(z,0)$,$\tilde{P}_1(z,0)$, and $\sum_{l=1}^{\infty}\tilde{V}_l(z,0)$, the following sequence of operations is performed.

Multiplying the Eq. (1) by z^0, and Eqs. (9)–(15), by z^n, taking summation from $n = 0$ to ∞ and using (16), we get

$$\begin{aligned}(\theta-\lambda)\tilde{P}_0(z,\theta) = P_0(z,0) - \tilde{R}(\theta)(P_1(z,0) - P_{1,0}(0) + \lambda z^2\tilde{P}_1(z,0))\\ + \sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0))\end{aligned} \tag{18}$$

$$(\theta-\lambda)\tilde{P}_1(z,\theta) = P_1(z,0) - (\lambda P_{00} + (1/z)P_0(z,0) + \lambda\tilde{P}_0(z,0))\tilde{S}(\theta) \tag{19}$$

$$(\theta-\lambda+\lambda z)\tilde{V}_1(z,\theta) = V_1(z,0) - P_{1,0}(0)\tilde{V}(\theta) \tag{20}$$

$$(\theta-\lambda+\lambda z)\tilde{V}_l(z,\theta) = V_l(z,0) - V_{l-1,0}(0)\tilde{V}(\theta), \quad 2\le l\le M \tag{21}$$

Substituting $\theta = \lambda$ in Eqs. (18) and (19) we have,

$$P_0(z,0) = \tilde{R}(\lambda)(P_1(z,0) - P_{1,0}(0) + \lambda z^2\tilde{P}_1(z,0) + \sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0))) \tag{22}$$

$$P_1(z,0) = \tilde{S}(\lambda)(\lambda P_{00} + (1/z)P_0(z,0) + \lambda\tilde{P}_0(z,0)) \tag{23}$$

Substituting for $P_1(z, 0)$ from Eq. (23) into Eq. (22) we have

$$P_0(z,0) = \frac{\begin{array}{c}(\tilde{R}(\lambda)(\tilde{S}(\lambda)(\lambda P_{00} + \lambda\tilde{P}_0(z,0)) + \lambda\tilde{P}_0(z,0) - P_{1,0}(0)\\ + \lambda z^2\tilde{P}_1(z,0) + \sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0))))\end{array}}{(1 - (\tilde{R}_3(\lambda)\tilde{S}(\lambda)/z))} \tag{24}$$

Substituting for $P_0(z, 0)$ from Eq. (22) into Eq. (23) we have

$$P_1(z,0) = \frac{(\tilde{S}(\lambda)(\lambda P_{00} + (\tilde{R}(\lambda)/z)(\lambda z^2\tilde{P}_1(z,0) - P_{1,0}(0) + \sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0))) + \lambda\tilde{P}_0(z,0)))}{(1 - (\tilde{S}(\lambda)\tilde{R}(\lambda)/z))} \tag{25}$$

Substituting for $P_0(z, 0)$ and $P_1(z, 0)$ from Eqs. (24) and (25) into Eqs. (18) and (19) we have

$$(\theta - \lambda)\tilde{P}_0(z,\theta) = \frac{\begin{array}{l}(\tilde{R}(\lambda)(\tilde{S}(\lambda)(\lambda P_{00} + \lambda\tilde{P}_0(z,0)) - P_{1,0}(0) + \lambda z^2\tilde{P}_1(z,0) \\ + \sum_{l=1}^{M}(V_l(z,0) - V_{l,o}(0))) - \tilde{R}(\theta)(\tilde{S}(\lambda)(\lambda P_{00} + (\tilde{R}(\lambda)/z) \\ (-P_{1,0}(0) + \lambda z^2\tilde{P}_1(z,0) + \sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0)))))\lambda\tilde{P}_0(z,0) \\ - \tilde{R}(\theta)(-P_{1,0}(0) + \lambda z^2\tilde{P}_1(z,0) + \sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0))))\end{array}}{(1 - (\tilde{R}(\lambda)\tilde{S}(\lambda)/z))} \tag{26}$$

$$(\theta - \lambda)\tilde{P}_1(z,\theta) = \frac{\begin{array}{l}(\tilde{S}(\lambda)(\lambda P_{00} + (\tilde{R}(\lambda)/z0)(\lambda z^2\tilde{P}_1(z,0) - P_{1,0}(0) + \sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0))) + \lambda\tilde{P}_0(z,0)) \\ - (\tilde{S}(\theta)/z)(\tilde{R}(\lambda)(\tilde{S}(\lambda)(\lambda P_{00} + \lambda\tilde{P}_0(z,0)) - P_{1,0}(0) + \lambda z^2\tilde{P}_1(z,0) \\ + \sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0)))) - \tilde{S}(\theta)(\lambda P_{00} + \lambda\tilde{P}_0(z,0))(1 - (\tilde{R}(\lambda)\tilde{S}(\lambda)/z)))\end{array}}{(1 - (\tilde{R}(\lambda)\tilde{S}(\lambda)/z))} \tag{27}$$

Substituting $\theta = 0$ and solving Eqs. (26) and (27) for $\tilde{P}_0(z,0)$ and $\tilde{P}_1(z,0)$ we get

$$\tilde{P}_0(z,0) = \frac{\begin{array}{l}((\tilde{R}(\lambda) - 1)(\lambda\tilde{S}(\lambda)P_{00} - P_{1,0}(0) + \sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0))) \\ (((\tilde{R}(\lambda)\tilde{S}(\lambda)/z) - 1) - z\tilde{R}(\lambda)(\tilde{S}(\lambda) - 1)) \\ + (\lambda P_{00} + (\tilde{R}(\lambda)/z)P_{1,0}(0) + (\tilde{R}(\lambda)/z)\sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0))) \\ (\tilde{S}(\lambda) - 1)(z^2(\tilde{R}(\lambda) - 1)))\end{array}}{\begin{array}{l}((\lambda((\tilde{S}(\lambda)\tilde{R}(\lambda)/z) - 1) - \lambda\tilde{S}(\lambda)(\tilde{R}(\lambda) - 1)) \\ (((\tilde{R}(\lambda)\tilde{S}(\lambda)/z) - 1) - z\tilde{R}(\lambda)(\tilde{S}(\lambda) - 1)) \\ - (\tilde{S}(\lambda) - 1)\lambda z^2(\tilde{R}(\lambda) - 1))\end{array}} \tag{28}$$

$$\tilde{P}_1(z,0) = \frac{\begin{array}{l}(\tilde{S}(\lambda)-1)(\tilde{R}(\lambda)-1)(\lambda\tilde{S}(\lambda)P_{00} - P_{1,0}(0) + \sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0))) \\ + ((((\tilde{R}(\lambda)\tilde{S}(\lambda))/z) - 1) - \tilde{S}(\lambda)(\tilde{R}(\lambda)-1))(\tilde{S}(\lambda)-1) \\ (\lambda P_{00} - (\tilde{R}(\lambda)/z)P_{1,0}(0) + (\tilde{R}(\lambda)/z)\sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0)))\end{array}}{\begin{array}{l}(\lambda(((\tilde{R}(\lambda)\tilde{S}(\lambda))/z) - 1) - \lambda\tilde{S}(\lambda)(\tilde{R}(\lambda)-1)) \\ ((((\tilde{R}(\lambda)\tilde{S}(\lambda))/z) - 1) - z\tilde{R}(\lambda)(\tilde{S}(\lambda)-1)) \\ - (\lambda z^2)(\tilde{R}(\lambda)-1)(\tilde{S}(\lambda)-1)\end{array}} \tag{29}$$

Substituting $\theta = \lambda - \lambda z$, in Eqs. (20) and (21) we have

$$\tilde{V}_1(z,0) = P_{1,0}(0)(\tilde{V}(\lambda - \lambda z) - 1) \tag{30}$$

$$\tilde{V}_l(z,0) = \frac{V_{l-l,0}(0)(\tilde{V}(\lambda - \lambda z) - 1)}{(-\lambda + \lambda z)}, \quad 2 \le l \le M \tag{31}$$

From Eqs. (30) and (31) we have

$$\sum_{l=1}^{M} \tilde{V}_l(z,0) = \frac{(P_{1,0}(0) + \sum_{l=1}^{M-1}\tilde{V}_{l,0}(0))(\tilde{V}(\lambda - \lambda z) - 1)}{(-\lambda + \lambda z)} \tag{32}$$

The following theorem is proved on substituting the expressions for $\tilde{P}_0(z,0)$,$\tilde{P}_1(z,0)$ and $\sum_{l=1}^{\infty}\tilde{V}_l(z,0)$ from Eqs. (28), (29) and (32) into Eq. (17).

Theorem 1 *The* PGF *P(z) of number of customers in the orbit is given by*

$$P(z) = \frac{(P_{00}M_3 + M_1 + M_2)(-\lambda + \lambda z) + (P_{1,0}(0) + \sum_{l=1}^{M-1} V_{l,0}(0))(\tilde{V}(\lambda - \lambda z) - 1)M_3}{M_3(-\lambda + \lambda z)} \tag{33}$$

where

$$\begin{aligned} M_1 = {} & (\tilde{R}(\lambda) - 1)(\lambda\tilde{S}(\lambda)P_{00} - P_{1,0}(0) + \sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0))) \\ & ((((\tilde{R}(\lambda)\tilde{S}(\lambda))/z) - 1) - z\tilde{R}(\lambda)(\tilde{S}(\lambda) - 1)) + (\tilde{S}(\lambda) - 1) \\ & (\lambda P_{00} - (\tilde{R}(\lambda)/z)P_{1,0}(0) + (\tilde{R}(\lambda)/z)\sum_{l=1}^{M}(V_l(z,0) - V_{l,0}(0)))(z^2(\tilde{R}(\lambda) - 1)) \end{aligned} \tag{34}$$

$$\begin{aligned}M_2 = {} & (\tilde{R}(\lambda)-1)(\lambda\tilde{S}(\lambda)P_{00}-P_{1,0}(0)+\sum_{l=1}^{M}(V_l(z,0)-V_{l,0}(0)))(\tilde{S}(\lambda)-1)\\ & +((((\tilde{R}(\lambda)\tilde{S}(\lambda))/z)-1)-z\tilde{R}(\lambda)(\tilde{S}(\lambda)-1))(\tilde{S}(\lambda)-1)(\lambda P_{00}-(\tilde{R}(\lambda)/z)P_{1,0}(0)\\ & +(\tilde{R}(\lambda)/z)\sum_{l=1}^{M}(V_l(z,0)-V_{l,0}(0)))\end{aligned} \tag{35}$$

$$\begin{aligned}M_3 = {} & (\lambda(((\tilde{R}_3(\lambda)\tilde{S}(\lambda))/z)-1)-\lambda\tilde{S}(\lambda)(\tilde{R}(\lambda)-1))((((\tilde{R}(\lambda)\tilde{S}(\lambda))/z)-1)\\ & -z\tilde{R}(\lambda)(\tilde{S}(\lambda)-1))-(\tilde{S}(\lambda)-1)\lambda z^2((\tilde{R}(\lambda)-1))\end{aligned} \tag{36}$$

4.2 Computational aspects of unknown probabilities

In this section, the unknown probabilities are expressed in terms of known constant P_{00}.

Theorem 2 *The unknown probabilities $P_{1,0}(0)$, $\sum_{l=1}^{M-1} V_{l,0}(0)$ and the unknown function $\sum_{l=1}^{M}(V_l(z,0)-V_{l,0}(0))$ are expressed in terms of known constant P_{00} as*

$$P_{1,0}(0) = \frac{\lambda P_{00}}{\tilde{V}^M(\lambda)}$$

$$\sum_{l=1}^{M-1} V_{l,0}(0) = \frac{\lambda P_{00}\,(1-\tilde{V}^{M-1}(\lambda))}{\tilde{V}^{M-1}(\lambda)\,(1-\tilde{V}(\lambda))}$$

$$\sum_{l=1}^{M}(V_l(z,0)-V_{l,0}(0)) = \lambda P_{00}\left(\frac{(\tilde{V}(\lambda-\lambda z)-1)\,(1-\tilde{V}^{M-1}(\lambda))}{\tilde{V}^{M-1}(\lambda)\,(1-\tilde{V}(\lambda))}+\frac{\tilde{V}(\lambda-\lambda z)}{\tilde{V}^M(\lambda)}-1\right)$$

where P_{00} is the probability that the server is idle and the number of customers in the orbit is zero.

Proof Substituting $\theta=\lambda$ in Eqs. (12) and (14) and after some algebra we have

$$V_{1,0}(0) = P_{1,0}(0)\ \tilde{V}(\lambda) \tag{37}$$

$$V_{M,0}(0) = V_{1,0}(0)\ \tilde{V}^{M-1}(\lambda) \tag{38}$$

Substituting for $V_{M,0}(0)$, $V_{1,0}(0)$ respectively from Eqs. (38) and (37) into Eq. (1) we have

$$P_{1,0}(0) = \frac{\lambda P_{00}}{\tilde{V}^M(\lambda)} \tag{39}$$

Now, using Eqs. (37–39) and after some algebra we have

$$\begin{aligned}\sum_{l=1}^{M-1} V_{l,0}(0) &= V_{1,0} + V_{1,0}\tilde{V}(\lambda) + V_{1,0}\tilde{V}^2(\lambda) + \cdots + V_{1,0}\tilde{V}^{M-2}(\lambda)\\ &= V_{1,0}\,\frac{(1-\tilde{V}^{M-1}(\lambda))}{(1-\tilde{V}(\lambda))}\\ &= \frac{\lambda P_{00}\,(1-\tilde{V}^{M-1}(\lambda))}{\tilde{V}^{M-1}(\lambda)\,(1-\tilde{V}(\lambda))}\end{aligned}$$

Now,

$$\begin{aligned}\sum_{l=1}^{M}(V_l(z,0)-V_{l,0}(0)) &= \sum_{l=1}^{M} V_l(z,0) - \sum_{l=1}^{M} V_{l,0}(0)\\ &= V_1(z,0) + \sum_{l=2}^{M} V_l(z,0) - \sum_{l=1}^{M} V_{l,0}(0)\end{aligned} \tag{41}$$

Substituting for $V_1(z,0)$ and $V_l(z,0)(2 \le l \le M)$ from Eqs. (20) and (21) into Eq. (41) we have

$$\sum_{l=1}^{M}(V_l(z,0)-V_{l,0}(0)) = (\tilde{V}(\lambda-\lambda z)-1)\sum_{l=1}^{M-1} V_{l,0}(0) + \tilde{V}(\lambda-\lambda z)P_{1,0}(0) - \lambda P_{00} \tag{42}$$

Substituting for $\sum_{l=1}^{M} V_{l,0}(0)$ and $P_{1,0}(0)$ from Eqs. (40) and (39) respectively into Eq. (42) we have

$$\sum_{l=1}^{M}(V_l(z,0)-V_{l,0}(0)) = \lambda P_{00}\left(\frac{(\tilde{V}(\lambda-\lambda z)-1)(1-\tilde{V}^{M-1}(\lambda))}{\tilde{V}^{M-1}(\lambda)\,(1-\tilde{V}(\lambda))} + \frac{\tilde{V}(\lambda-\lambda z)}{\tilde{V}^{M}(\lambda)} - 1\right) \tag{43}$$

Hence the theorem is proved. □

5 Stability condition

The PGF P(z) has to satisfy the condition $\lim_{z\to 1} P(z) = 1$. In order to satisfy this condition L'Hospital's rule is applied to Eq. (33). Since P_{00}, $P_{1,0}(0)$ and $\sum_{l=1}^{M} V_{l,0}(0)$ are probabilities the numerator of P(z) is positive when $z \to 1$. So $\lim_{z\to 1} P(z) = 1$ is satisfied if $\tilde{S}(\lambda)\,\tilde{R}(\lambda)(1-\tilde{R}(\lambda)-2\tilde{S}(\lambda)) + 2(\tilde{R}(\lambda)+\tilde{S}(\lambda) -$

$1) > 0$ which is the condition to be satisfied for the existence of steady state for the model under consideration.

To prove the necessary and sufficient condition for the system to be stable, we study the ergodicity of the embedded Markov chain at the customers' departure/collision occurring epochs. Let $\{t_k : k \in z_+\}$ be the sequence of epochs of either the service completion or collision occurring times at which the server become idle and $X_k = X(t_k+)$ be number of customers in the orbit immediately service completion time or collision occurring time.

Theorem 3 $\{X_k; k \in Z_+\}$ *be an embedded Markov Chain which is ergodic iff* $2\left(1 - \tilde{S}(\lambda)\right) < \tilde{R}(\lambda)$.

Proof By Foster's criterion Pakes (1969), we can prove that the irreducible and aperiodic Markov chain $\{X_k; k \in Z_+\}$ is positive recurrent. Let $f(n), n \in Z_+$ and $\epsilon > 0$ be a nonnegative function such that the mean drift $C_n = E\left[f(X_{k+1}) - f(X_{k]})_{/X_k=n}\right]$ is finite for all $n \in Z_+$. Here $C_n =$

$$\begin{cases} 2\left(1 - \tilde{S}(\lambda)\right) - \tilde{R}(\lambda), & n = 1, 2, 3, \ldots \\ 2\left(1 - \tilde{S}(\lambda)\right), & n = 0 \end{cases}$$

Since $2\left(1 - \tilde{S}(\lambda)\right) < \tilde{R}(\lambda)$ we have $\lim_{n\to\infty} C_n < 0$. This shows that $X_k; k \in Z_+$ is positive recurrent. The term $\left(1 - \tilde{S}(\lambda)\right)$ implies that the arriving primary customer enters into the orbit and also $\left(1 - \tilde{S}(\lambda)\right)$ implies that the arriving primary customer proceeds to the server and collides with the customer in service resulting in both being transformed to the orbit. So, the necessary and sufficient condition for the stability is $2\left(1 - \tilde{S}(\lambda)\right) < \tilde{R}(\lambda)$. □

6 Performance characteristics

In this section, certain useful performance measures of the proposed model like, expected number of customers in the orbit, expected length of busy period, expected length of busy cycle, probability that the server is idle, probability that the server is busy and the probability that the server is on vacation are derived.

6.1 The mean number of customers in the orbit

The expected number of customers in the orbit is derived using PGF given in Eq. (33) and $L_Q = \lim_{Z\to 1} \frac{d}{dz} P(z)$. Since the expression for P(z) is too large the numerical values of L_Q are calculated using the software mathematica.

6.2 Expected length of busy period

In this section, we consider a busy period of the system for the model under discussion. The system busy period T_b starts at an epoch when an arriving customer finds an empty system and ends at the next departure epoch at which the system is empty. Using this, the mean length of the system busy period of this model is directly obtained by the theory of regenerative processes which leads to the following limiting probability

$$\begin{aligned} P_{00} &= P(C(t) = 0, N(t) = 0) \\ &= \frac{E(T_{00})}{\frac{1}{\lambda} + E(T_b)} \end{aligned}$$

where T_{00} is the amount of time in a regenerative cycle during which the system is in the state (0, 0). It is clear that $E(T_{00}) = \frac{1}{\lambda}$ so that $E(T_b) = \frac{1}{\lambda}\left(\frac{1}{P_{00}} - 1\right)$. Hence, if T_b is the length of busy period, then under the steady state condition, the expected length of busy period is $E(T_b) = \frac{1}{\lambda}\left(\frac{1}{P_{00}} - 1\right)$ where P_{00} is obtained from Eq. (33) using the condition $\mathrm{Lim}_{z\to 1} P(z) = 1$.

6.3 Expected length of busy cycle

If T_c is the length of busy cycle, then under the steady state conditions and by the argument of alternating renewal process, the expected length of busy cycle $E(T_c)$ is obtained as

$$\begin{aligned} E(T_c) &= \text{Expected length of busy period} + \text{Expected length of idle period.} \\ &= E(T_b) + \frac{1}{\lambda} \\ &= \frac{1}{\lambda}\frac{1}{P_{00}} \end{aligned}$$

6.4 Probability that the server is idle

Let I be the idle period random variable and let P(I) be the probability that the server is idle at time t. Using Eq. (28) and applying limit $z \to 1$ we get the probability that the server is idle as

$$\begin{aligned} P(I) &= P_{00} + \tilde{P}_0(1,0) \\ &= E(V)\,(\tilde{R}(\lambda) - 1)\left(\sum_{l=1}^{M-1} V_{l,0}(0) + P_{1,0}(0)) + P_{00}\tilde{R}(\lambda)(1 - 2\,\tilde{S}(\lambda)\right) \end{aligned}$$

6.5 Probability that the server is busy

Let B be the busy period random variable and P(B) be the probability that the server is busy at time t. Using Eq. (29) and applying limit $z \to 1$ we get the probability that the server is busy as

$$P(B) = \tilde{P}_1(1,0) = \frac{(\tilde{S}(\lambda) - 1)\,(E(V)\,(\sum_{l=1}^{M-1} V_{l,0}(0) + P_{1,0}(0)) + P_{00}\tilde{R}(\lambda))}{(2 - z\,\tilde{S}(\lambda) - \tilde{R}(\lambda))}$$

6.6 Probability that the server is on vacation

Let V be the vacation time random variable and P(V) be the probability that the server is on vacation at time t. Using Eq. (32) and applying limit $z \to 1$ we get the probability that the server is on vacation as

$$P](V) = \sum_{l=1}^{M} \tilde{V}_l(1,0) = E(V)\left(\sum_{l=1}^{M-1} V_{l,0}(0) + P_{1,0}(0)\right)$$

7 Special cases

In this section, some special cases of the proposed model by specifying the service time, vacation time and retrial time random variables as K-Erlang, exponential and hyper exponential distribution are discussed.

Case (i) M/G/1 retrial queue with general retrial time, modified M-vacations and collision with Erlangian vacation time.

If the vacation time is assumed to be K-Erlang with probability density function, $v(x) = \frac{(ku)^k x^{k-1} e^{-kux}}{(k-1)!}, k > 0$ where u is the parameter, then

$$\tilde{V}(\lambda - \lambda z) = (uk/(uk + \lambda(1 - z)))^k \tag{44}$$

Substituting (44) in (33), the PGF of the M/G/1 retrial queue with general retrial time, modified M-vacations and collision is given as

$$P(z) = \frac{(P_{00}M_3 + M_1 + M_2)(-\lambda + \lambda z) + (P_{1,0}(0) + \sum_{l=1}^{M-1} V_{l,0}(0))\,((uk/(uk + \lambda(1 - z)))^k - 1)M_3}{M_3(-\lambda + \lambda z)}$$

where M_1, M_2, and M_3 are given by Eqs. (34), (35) and (36) respectively with

$$\sum_{l=1}^{M-1} V_{l,0}(0) = \frac{\lambda P_{00}\ (1 - ((uk/(uk + \lambda))^k)^{M-1})}{((uk/(uk + \lambda))^k)^{M-1}\,(1 - (uk/(uk + \lambda))^k)}$$

and

$$\sum_{l=1}^{M}(V_l(z,0)-V_{l,0}(0)) = \lambda P_{00}\left(\frac{((uk/(uk\ +\ \lambda(1-z)))^k-1)\ (1-((uk/(uk\ +\ \lambda))^k)^{M-1})}{((uk/(uk\ +\ \lambda))^k)^{M-1}\ (1-(uk/(uk\ +\ \lambda))^k)}\right) + \lambda P_{00}\left(\frac{(uk/(uk\ +\ \lambda(1-z)))^k}{((uk/(uk\ +\ \lambda(1-z)))^k)^M}-1\right)$$

Case (ii) M/G/1 retrial queue with general retrial time, modified M-vacations and collision with Exponential vacation time.

If the vacation time is assumed to be exponential with probability density function $v(x) = ue^{-ux}$ where u is the parameter, then

$$\tilde{V}(\lambda-\lambda z) = (u/(u+\lambda(1-z))) \tag{45}$$

Substituting (45) in (33), the PGF of the M/G/1 retrial queue with general retrial time, modified M-vacations and collision is given by

$$P(z) = \frac{(P_{00}M_3+M_1+M_2)(-\lambda+\lambda z)\ +\ (P_{1,0}(0)+\sum_{l=1}^{M-1}V_{l,0}(0))\,((u/(u+\lambda(1-z)))-1)M_3}{M_3(-\lambda+\lambda z)}$$

where M_1, M_2, and M_3 are given by Eqs. (34), (35) and (36) respectively with

$$\sum_{l=1}^{M-1}V_{l,0}(0) = \frac{\lambda P_{00}\ (1-((u/(u\ +\ \lambda)))^{M-1})}{((u/(u\ +\ \lambda)))^{M-1}(1-(u/(u\ +\ \lambda)))}$$

and

$$\sum_{l=1}^{M}(V_l(z,0)-V_{l,0}(0)) = \lambda P_{00}\left(\frac{((u/(u\ +\ \lambda(1-z)))-1)\ (1-((u/(u\ +\ \lambda)))^{M-1})}{((u/(u\ +\ \lambda)))^{M-1}\ (1-(u/(u\ +\ \lambda)))}\right) + \lambda P_{00}\left(\frac{(u/(u\ +\ \lambda(1-z)))}{((u/(u\ +\ \lambda(1-z))))^M}-1\right)$$

Case (iii) M/G/1 retrial queue with general retrial time, modified M-vacations and collision with Hyper Exponential vacation time.

If the vacation time is assumed to be hyper exponential with probability density function $v(x) = cu\,e^{-ux} + (1-c)\,w\,e^{-wx}$ where x, u, w > 0 and $0 \le c \le 1$, then

$$\tilde{V}(\lambda-\lambda z) = (uc/(u\ +\ (\lambda-\lambda z)))\ +\ (w(1-c)/(w\ +\ (\lambda-\lambda z))) \tag{46}$$

Substituting (46) in (33), the PGF of the M/G/1 retrial queue with general retrial time, modified M-vacations and collision is given by

$$P(z) = \frac{((P_{00}M_3 + M_1 + M_2)(-\lambda + \lambda z) + (P_{1,0}(0) + \sum_{l=1}^{M-1} V_{l,0}(0))((uc/(u + (\lambda - \lambda z))) + (w(1-c)/(w + (\lambda - \lambda z))) - 1)M_3)}{M_3(-\lambda + \lambda z)}$$

8 Numerical results

In this section, some numerical results are provided to justify the theoretical results obtained. To study the effect of various parameters on the system performance measures, the following notations are used and some assumptions are made:

Average arrival rate	λ
Service time distribution is exponential with parameter	μ
Vacation duration is exponential or Erlang-2 with parameter	η
Retrial rate	γ
Number of vacations (modified vacations)	M

Table 1 and Fig. 2 represent the effect of arrival rate λ on the mean orbit size L_Q for $M = 4$. And also Table 1 shows the way in which the expected busy period and expected busy cycle changes for different values of arrival rate λ. It is assumed that $\mu = 0.7$, $\eta = 0.5$, $\gamma = 0.7$. From the table and the figure, the following observations can be made.

- As arrival rate increases, the mean orbit size increases.

Table 1 Arrival rate λ (vs) mean orbit size L_Q, expected busy period (EBP) and expected busy cycle (EBC) for $M = 4$

λ	M = 4		
	L_Q	EBP	EBC
0.1	0.2091	1.4793	1.5793
0.11	0.2359	1.7834	1.8934
0.12	0.2646	2.1182	2.2382
0.13	0.2954	2.4858	2.6158
0.14	0.3289	2.8886	3.0289
0.15	0.3657	3.3302	3.4802
0.16	0.4066	3.8149	3.9749
0.17	0.4526	4.3482	4.5182
0.18	0.5052	4.9374	5.1174
0.19	0.5658	5.5917	5.7817
0.20	0.6367	6.3231	6.5231

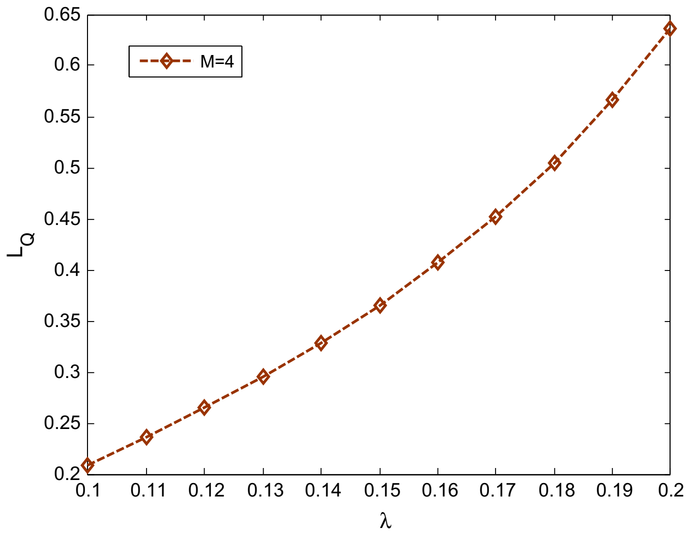

Fig. 2 Arrival rate λ (vs) mean orbit size L_Q for M = 4

Table 2 Vacation parameter η (vs) mean orbit size L_Q, expected busy period (EBP) and Expected busy cycle (EBC) for M = 4

η	Exponential			Erlang-2		
	L_Q	EBP	EBC	L_Q	EBP	EBC
1.0	3.9905	7.3590	7.6590	0.9104	17.378	17.678
1.1	4.0250	6.3610	6.6620	0.9559	11.728	12.028
1.2	4.0548	5.6128	5.9128	1.0187	8.2026	8.5026
1.3	4.0803	5.0337	5.3337	1.0991	5.8978	6.1978
1.4	4.1022	4.5751	4.8751	1.1973	4.3319	4.6319
1.5	4.1209	4.2044	4.5044	1.3142	3.2329	3.5329
1.6	4.1368	3.8995	4.1995	1.1416	2.4403	2.7403
1.7	4.1405	3.6449	3.9449	1.6117	1.8550	2.1550

- As the arrival rate increases, the expected busy period and expected busy cycle also increase.

Table 2 gives the effect of vacation parameter on the mean orbit size L_Q, expected busy period and expected busy cycle. In Fig. 3, the mean orbit size L_Q is compared for different arrival rates. The vacation times are considered as exponential and Erlang-2 with parameters $\lambda = 0.3$, $\mu = 0.7$, $\gamma = 0.7$, $M = 4$. From the table and the figure, the following points are observed.

- As the vacation rate increases, the mean orbit size increases.
- As the vacation rate increases, the expected busy period and expected busy cycle decrease.

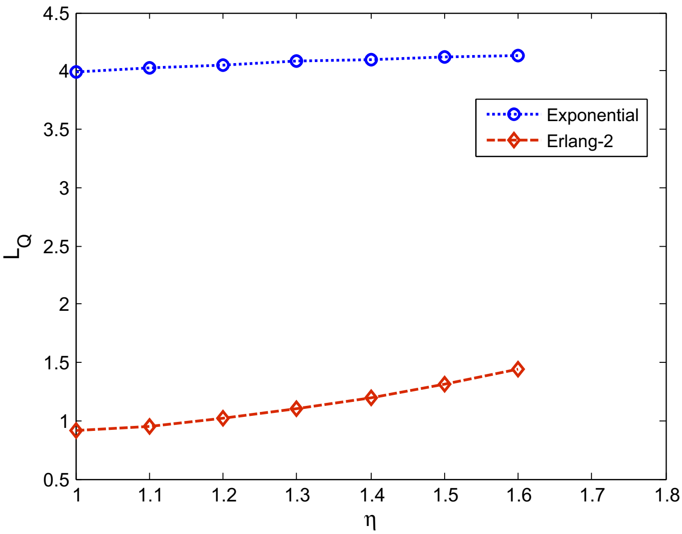

Fig. 3 Vacation parameter η (vs) mean orbit size L_Q

9 Conclusion

In this chapter an M/G/1 retrial queueing system with general retrial time, modified M-vacations and collision is analyzed. The PGF for the queue size at an arbitrary time epoch has been derived. Some system performance measures, such as mean orbit size, expected length of busy period, expected length of busy cycle, probability that the server is idle, probability that the server is busy and the probability that the server is on vacation are obtained. The theoretical development of the model is justified with the numerical results.

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