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Antenna selection and adaptive power allocation for IA-based underlay CR

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Abstract: In underlay spectrum sharing cognitive radio (CR), the secondary users (SUs) share the spectrum with the primary users (PUs) in such a way that the interference caused by the SUs should be less than the interference temperature (IT) threshold. Interference Alignment (IA) is considered as a novel interference management technique for CR networks (CRN) which eliminates the interference at the PUs and also the interference among the SUs. However, most of the iterative solutions for IA aimed at removing the interference between the users, does not take into account the QoS (Quality of Service) of the PUs and SUs. Also the achievable rate of PUs by employing IA is less than that of the maximum achievable rate in a MIMO (Multiple input multiple output) system with no interference. In this paper we first present an Antenna Selection (AS) scheme to significantly improve the QoS of PUs and then we have proposed an adaptive power allocation (PA) scheme which aims at optimum PA in view of the proportional fairness among SUs. Numerical results are derived for comparing the proposed schemes in terms of outage performance of PUs and SUs. Simulation results show the effectiveness of the proposed methods in terms of SNR.

1 Introduction

With the rapid deployment of wireless communication technologies, large bandwidth is needed to offer high data rate services. However, due to the limited availability of unallocated spectrum, the spectrum shortage problem is found to be increased. Cognitive radio (CR) technology is known as an efficient technique to dynamically utilise the scarce wireless resource. The main concept behind the CR is to permit the unauthorised users/ secondary users (SUs) to transmit their data in a spectrum area licenced to the authorised users/primary users (PUs) of the network [1]. Multiple-input–multiple-output (MIMO) has been considered as a promising candidate for CR system. MIMO helps in increasing the throughput and reducing the Bit Error Rate (BER) of the system [2, 3].

There are three different approaches for spectrum access in CR networks (CRNs): interweave, overlay and underlay. In underlay scenario, which is the focus of this paper, the PUs share the spectrum with the SUs. Thus, concurrent transmissions are possible in underlay mode without the need for spectrum sensing. However, the SUs should ensure that the interference caused by them to the PUs should be below the interference temperature limit [4]. Therefore, it is necessary to device proper signal processing procedures to control the interference at the primary receivers (PRXs). Furthermore, in order to ensure the quality of service (QoS) of the SUs, the interference from other SUs and PUs should be reduced at the secondary RXs.

Interference alignment (IA) is an emerging interference management technique, which exploits the precoding matrices to constrain all the interference from different users into one half of the signal space at each RX and leave the other half without interference for the desired signal. Then, by using interference suppression decoding matrices, the desired signal can be recovered free of interference [5]. The feasibility conditions of IA are studied in [6]. Two iterative, distributed IA algorithms, minimum interference leakage (MinIL) and maximum signal-to-interference noise ratio (MaxSINR) are designed in [7] to perform IA in wireless interference networks. These algorithms utilise the reciprocity of wireless networks, thus eliminating the need for global channel knowledge.

Meanwhile IA techniques can be used in underlay scenario in CR, where the PUs and SUs simultaneously transmit over the same frequency or time [7–11]. In [8], Du *et al.* consider IA in peer-to-peer underlay CR, which is based on solving an optimisation problem, to minimise the total IL, while limiting the interference level at the PURX. Another work in [9] is employed to find an approximation algorithm for IA, by taking into account the practical issues in implementing the IA algorithm. In [10], Rezai and Tadaion propose a new method for IA considering multiple PUs and multiple SUs. They employ separate IA procedure for PUs; moreover, when the SUs enter the PU network, the precoding matrices for SUs are separately designed to reduce the distance between the existing PU interference subspace and the new SU's interference space at the PRX. However, as the number of SUs increase, the complexity of the IA procedure also increases. Furthermore, to maximise the rates of PUs and SUs water-filling (WF) power allocation (PA) is performed. The authors in [11] proposed an optimal transceiver design scheme for underlay CR and they present two partial IA (PIA) algorithms [PIA considering interference between SUs (PIA-SU) and PIA not considering interference between SUs (PIA-nSU)], for deriving the precoding matrices for SUs. They concluded that the performance of the second algorithm, i.e. PIA-nSU is better than the first algorithm and also it imposes less overhead and complexity. All these works [7–11] aim at deriving the IA procedure for CRNs; on the other hand, the QoS of PUs/SUs are not addressed in these literatures.

Antenna selection (AS) is usually utilised in MIMO systems to improve the QoS and the reliability of reception [12]. Employing AS at the transmitter (TX) and/or at the RX side reduces the hardware complexity, by decreasing the number of Radio Frequency (RF) chains, while preserving the diversity benefits and gain rate. Considering the superior performances of AS, these techniques in the context of IA were studied in different literatures to improve sum rate and error-rate performance of the MIMO system [13–15]. Different selection criteria for AS in MIMO IA network were studied in [13]. They suggested two better measures: based on the eigenvalues of the effective channel matrices and on the RX side signal-to-NR (SNR). Recently, He *et al.* [14] studied the received SINR degradation problem in IA-based CRNs and they proposed new methods based on the idea of exploiting

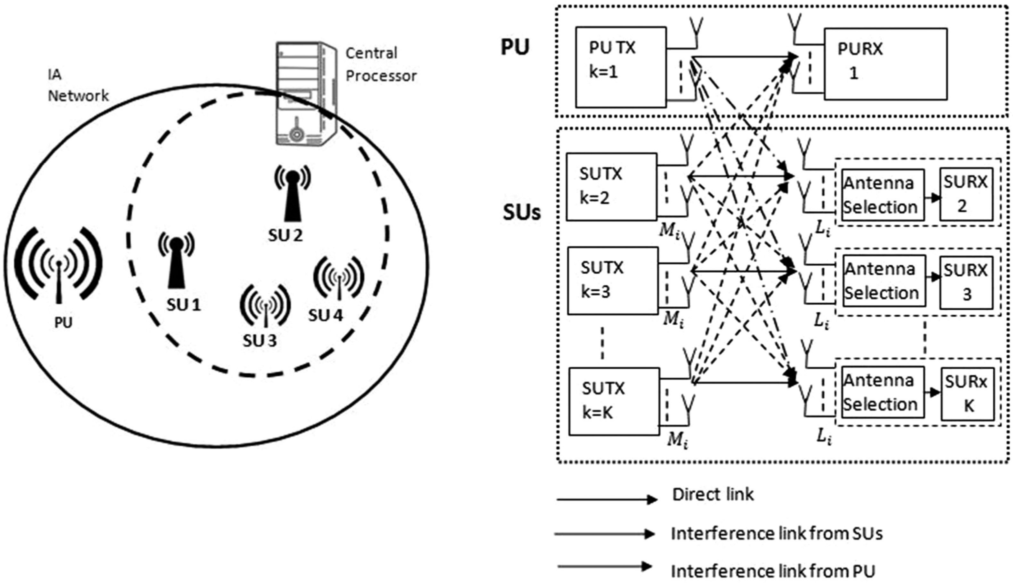

Fig. 1 *System model of MIMO CR*

multiuser diversity and antenna diversity. These AS methods guarantee the performance of PUs. However, the performance or the QoS improvement of SUs is not addressed in these literatures.

PA in CRNs aim at efficiently assigning transmission power in order to maintain the communication quality at a satisfactory level for PUs and SUs. In the early research works, PA for SUs is carried out in underlay spectrum sharing CRNs such that the interference power to (PU) should be below the IT threshold [16]. Furthermore, those studies which aim at deriving IA algorithms for underlay spectrum sharing CRNs consider only equal PA and do not take into account the QoS of PUs or SUs. Li *et al.* [15] proposed different AS methods to improve the QoS of the PU and SUs, but adapted only equal PA. Most recently, in [17] PA and control were adapted to IA-based CRN to improve the QoS of SUs. They proposed three different PA algorithms with different aims, but the QoS improvement of PUs is not explored up to the level.

In this paper, we first analyse the performance of several IA schemes for underlay CRNs found in literatures. Since the QoS of PUs and SUs are not considered in these IA algorithms, we propose an AS scheme followed by PA scheme to improve the QoS of PUs as well as SUs. The AS is done particularly to maximise the rate of PUs and to reduce the SINR degradation problem of IA algorithms. The optimal WF PA algorithm allocates more power to users in good channel condition and less or even zero power to the users having worst channels, with an aim to maximise the total rate under the constraint of total transmission power. Thus, the overall performance gets deteriorated in view of fairness, which requires, guaranteeing a certain level of transmission rate to users with low average channel gains. In this context, we propose adaptive PA strategy which considers proportional fairness among the users and maximises the QoS of SUs as well as PUs. Thus, our proposed method aims at guaranteeing the minimum rate requirements of all users as well as provides the fairness in PA among the SUs.

The remainder of this paper is organised as follows: Section 2 describes the system model and comparison of distributed iterative IA algorithms followed by the problem statement. The proposed AS method and PA scheme is discussed in Section 3. Section 4 presents the simulation results, comparing the performance of the proposed schemes which is followed by conclusion of this work in Section 5.

Notations: Matrices and Vectors are denoted by boldface uppercase and boldface lowercases, respectively. $\| \boldsymbol{A} \|$, $\| \boldsymbol{v} \|_2$ represents frobenius norm of a matrix $\boldsymbol{A}$ and ℓ^2-norm of a vector $\boldsymbol{v}$, respectively. $\boldsymbol{I}_M$ represents $M \times M$ identity matrix. $\mathbb{E}[\,.\,]$ represents the statistical expectation operator and $\mathbb{P}\{\,.\,\}$ represents probability. det $(\,.\,)$, Tr$(\,.\,)$, $(\,.\,)^{-1}$ and $(\,.\,)^{\dagger}$ denote the determinant, trace, inverse and conjugate transpose operations. rank$(\boldsymbol{A})$, $\boldsymbol{\Upsilon}_d(\boldsymbol{A})$ denote the rank, left singular vector matrix of the reduced Singular Value Decomposition (SVD) of matrix $\boldsymbol{A}$, respectively. diag$([\boldsymbol{x}],\ 0)$ represents diagonal matrix with vector $\boldsymbol{x}$ along the diagonal.

2 System model and problem statement

In this section, we present the system model for IA-based underlay CRNs and the performance comparisons of the different IA schemes. Then, we discussed the QoS requirements of both PUs and SUs in underlay CRN and we present the problem statement for satisfying the QoS of the PU as well as the SU in IA-based CRN.

2.1 System model

Consider a K user MIMO CRN with 1 PU and $(K-1)$ SUs, where the index $'1'$ represents the PU and the remaining $(K-1)$ users are SUs, depicted in Fig. 1. Each TX and RX is equipped with M_i TX antennas and N_i RX antennas, respectively. All the SUs share the spectrum with the PU simultaneously. For convenience, we define each user as a TX–RX pair, where the TX communicates with the RX. For a PU, we call the TX and the RX as PUTX and PURX, respectively. Similarly, we define the SUTX and SURX. The received signal at the ith RX is

$$\boldsymbol{y}_i = \boldsymbol{H}_{ii}\boldsymbol{s}_i + \sum_{j=0,\ j \neq i}^{K} \boldsymbol{H}_{ij}\boldsymbol{s}_j + \boldsymbol{z}_i \tag{1}$$

$\boldsymbol{H}_{ij}$ is the $N_i \times M_j$ channel coefficient matrix between the j^{th} TX and the ith RX, where $i,\ j \in \{1,\ \dots,\ K\}$. All the entities of $\boldsymbol{H}_{ij}$ are independent and identically distributed and follow Gaussian distribution with mean zero and unit variance $[\mathcal{N}(0,\ 1)]$. $\boldsymbol{z}_i$ is the $N_i \times 1$ additive white Gaussian noise vector of the ith RX with mean 0 and covariance $\sigma_i^2 \boldsymbol{I}_{N_i}$. $\boldsymbol{s}_j$ is the transmitted symbols of the j^{th} user with power $\mathbb{E}\left[\| \boldsymbol{s}_j \|_2^2\right] = P_j$. We assume block fading model, i.e. the channel matrices are constant during each time slot and change independently between time slots. Also, we assume that the channel coefficients are known at all TXs and RXs. The second term in (1) corresponds to interference signal from other TXs.

IA is used to manage and control the interference in underlay CRNs. The received signal at the ith RX after IA is

$$\tilde{\boldsymbol{Y}}_i = \boldsymbol{U}_i^{\dagger}\boldsymbol{H}_{ii}\boldsymbol{V}_i\boldsymbol{s}_i + \sum_{j=1,\ j \neq i}^{K} \boldsymbol{U}_i^{\dagger}\boldsymbol{H}_{ij}\boldsymbol{V}_j\boldsymbol{s}_j + \boldsymbol{U}_i^{\dagger}\boldsymbol{z}_i \tag{2}$$

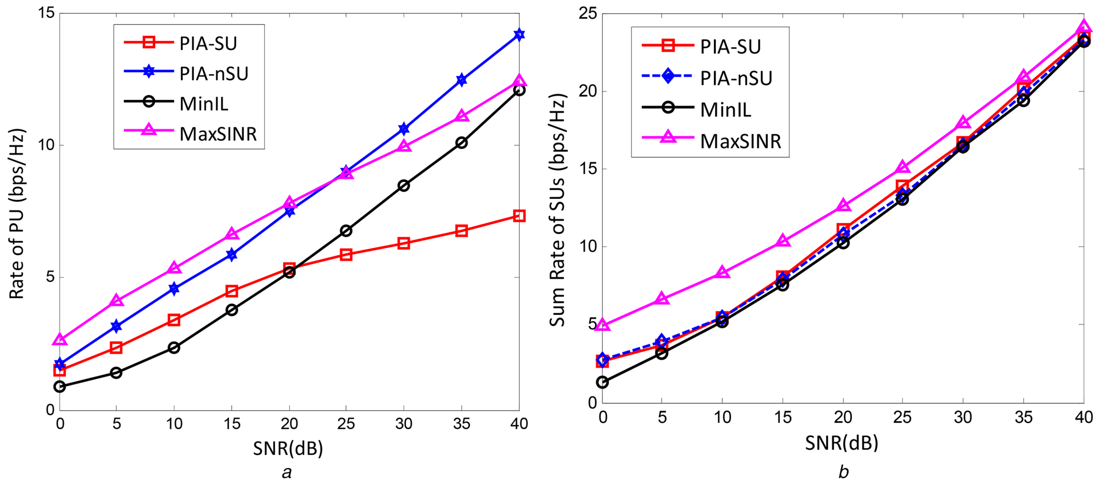

Fig. 2 *Comparison of Rate versus SNR of different IA algorithms*
(a) Rate of PU, ***(b)*** Sumrate of SUs
($K = 3$ (PU: 1, SU: 2), $M_i = 3$, $N_i = 3$, $d_i = 1$, $\sigma_i^2 = 1$, $\forall i = \{i = 1, \dots, K\}$)

where $\tilde{\boldsymbol{Y}}_i = \boldsymbol{U}_i^\dagger \boldsymbol{y}_i$ is the $d_i \times 1$ received signal vector and $\boldsymbol{s}_j$ is the $d_j \times 1$ transmitted signal vector. d_j is the number of data streams of user j. $\boldsymbol{V}_i$, $\boldsymbol{U}_i$ are the $M_i \times d_i$ precoding and $N_i \times d_i$ decoding matrices, respectively, for the ith user. To perfectly eliminate the interference, the $\boldsymbol{V}_j$ and $\boldsymbol{U}_i$ are designed such that the following conditions are met:

$$\boldsymbol{U}_i^\dagger \boldsymbol{H}_{ij} \boldsymbol{V}_j = 0, \quad \forall j \neq i \tag{3}$$

$$\operatorname{rank}\left(\boldsymbol{U}_i^\dagger \boldsymbol{H}_{ii} \boldsymbol{V}_i\right) = d_i \tag{4}$$

Equation (3) ensures that there is no interference from unintended TXs and (4) guarantees that the desired signal achieves d_i degrees of freedom.

2.2 Comparison of IA algorithms

Figs. 2*a* and *b* present the comparison of different IA algorithms (MaxSINR, MinIL [7], PIA-SU and PIA-nSU [11]) based on the achieved rate and sum rate of PUs and SUs, respectively. In both graphs, the MaxSINR algorithm outperforms the MinIL algorithm at low SNR, where noise dominates the interference and the gap gets reduced at high SNRs. This is because at high-SNR interference rather than the noise dominates. Comparing the SU sum rate, the MaxSINR algorithm achieves good sum rate when compared with all other algorithms. PIA-nSU algorithm gives better rates for PU when compared with PIA-SU algorithm. At high SNRs above 25 dB, PIA-nSU achieves higher PU rates than MaxSINR. This is because in PIA-nSU algorithm, the PUs precoding matrices are designed so as to improve its performance. In PIA-nSU, the precoding matrices for SUs are designed without considering other SUs. Therefore, the sum rates achieved by PIA-nSU for SUs are almost similar to other algorithms, except the MaxSINR. The PIA-nSU algorithm takes least overhead, lowest complexity when compared with other schemes [11].

2.3 Problem statement

In the remaining of this paper, we use MaxSINR and PIA-nSU IA algorithms to analyse the QoS for IA-based underlay CRN. IA focuses mainly on eliminating the interference without involving the QoS of desired signal. MaxSINR IA algorithm is based on maximising the SINR. However, when compared with the scenario of no SUs and a single PU with MIMO, its performance becomes low at high SNR. In general, an MIMO system without any IA and assuming that the channel has zero mean and unit variance, the maximum SINR achieved by it is

$$\mathrm{SINR}_{\max} = \frac{P_1}{\sigma^2} \lambda_{\max}\left(\boldsymbol{H}_{11}^\dagger \boldsymbol{H}_{11}\right) \tag{5}$$

In IA-based CRN, even though the interference gets eliminated effectively, the performance of PU gets degraded compared with that in the MIMO system (5). On the basis of the above analysis, the QoS degradation problem of PUs in IA-based CRN should be solved to make it more practical. PUs transmission rate can be considered as a parameter to measure the QoS of PUs. In this work, we employ AS technique to improve the QoS of PUs. The diversity gain obtained through AS significantly boosts the PUs performance especially at high SNRs. The transmission rate of the ith user is given by

$$R_i = \log_2\left(\left|\boldsymbol{I}_{d_i} + \frac{\boldsymbol{U}_i^\dagger \boldsymbol{Q}_i^1 \boldsymbol{U}_i}{\boldsymbol{U}_i^\dagger\left(\sigma^2 \boldsymbol{I}_{N_i} + \boldsymbol{Q}_i^2\right)\boldsymbol{U}_i}\right|\right) \tag{6}$$

where

$$\boldsymbol{Q}_i^1 = \frac{P_i}{d_i} \boldsymbol{H}_{ii} \boldsymbol{V}_i \boldsymbol{V}_i^\dagger \boldsymbol{H}_{ii}^\dagger \tag{7}$$

$$\boldsymbol{Q}_i^2 = \sum_{j=1,\ j \neq i}^{K} \frac{P_j}{d_j} \boldsymbol{H}_{ij} \boldsymbol{V}_j \boldsymbol{V}_j^\dagger \boldsymbol{H}_{ij}^\dagger \tag{8}$$

R_1 corresponds to PU rate and R_i, $i \in \{2, \dots, K\}$ are the rates of SUs. We limit the number of data streams to one throughout this paper, i.e. $d_i = 1$, $\forall i\{1, 2, \dots, K\}$. Hence, we define $h_i = \boldsymbol{u}_i^\dagger \boldsymbol{H}_{ii} \boldsymbol{v}_i$, $\boldsymbol{v}_i$ and $\boldsymbol{u}_i$ are unitary precoding and decoding vectors for the ith user, respectively. $|h_i|^2$ follows exponential distribution with unit mean and variance.

PA is an important candidate for IA-based CRNs to improve the QoS of PUs and SUs. In CRN, priority is given to PUs and in underlay scenario PUs share the spectrum with SUs. Thus, PUs always have some data to send. So, while performing PA, care should be taken that the minimum rate requirement of PUs should not be disturbed unless the total power available is less than the minimum power required. If $R_{\mathrm{th}}(1)$ is the minimum rate requirement (or threshold rate) of PU, then the minimum power required by PU (without considering interference from other users) is

$$P_{1-\min} = \frac{\left(2^{R_{\mathrm{th}}(1)} - 1\right)\sigma_1^2}{|h_1|^2} \tag{9}$$

Also, PUs involve in spectrum trading policy. It leases the spectrum to the SUs and it should maintain a certain QoS for SUs too. So, in order to guarantee the QoS of all SUs fairness-based PA should be employed to almost satisfy the minimum rate requirement of SUs. If $R_{\text{th}}(i)$, $\forall i = \{2, \ldots, K\}$ is the rate requirement of SUs, then the minimum power required by each SU is

$$P_{i-\min} = \frac{(2^{R_{\text{th}}(i)} - 1)\sigma_i^2}{|h_i|^2}, \quad \forall i = \{2, \ldots, K\} \tag{10}$$

We assume that the IT threshold of PU does not need to be considered explicitly in PA because IA perfectly eliminates the interference at all RXs. Therefore, the problem statement is formulated based on maximising the sum rate subjected to the power constraints without considering the interference constraints.

3 Proposed method

3.1 AS to improve QoS of PUs

AS technique is employed to improve the QoS of the PUs in IA-based CRN. Since PU has the priority in CRN, performing AS at primary is not proper. Therefore, AS is done at SURX, where some additional antennas are equipped at each SURX. Assume M_i, $i \in \{1, \ldots, K\}$ are the number of TX antennas for the ith user and L_i, $i \in \{2, \ldots, K\}$ are the number of RX antennas of SUs. Using AS we need to choose N_i, $i \in \{2, \ldots, K\}$ number of antennas from each SURX. Therefore, the total available solutions are $\prod_{i=2}^{K} \binom{N_i}{L_i}$. Let Φ denote the set of n possible antenna combinations $\Phi = \{\Omega_1, \Omega_2, \ldots, \Omega_n\}$. According to AS method, among these one set of antenna combination which satisfies most of the objectives are selected. Since we perform AS to improve the QoS of PU, the antenna subset is selected such that to maximise the PU SINR (equivalently to maximise the rate), subject to the condition that the rate of PU should be greater than $R_{\text{th}}(1)$. If none of the antenna combinations satisfy $R_{\text{th}}(1)$, then no SUs are allowed to share the spectrum with PU, in order to maintain the QoS of PU. Thus the objective function for AS is

(**P1**)

$$\begin{aligned} &\Omega^* = \arg \max_{\Omega \in \{\Phi\}} \{R_1\} \\ &\text{such that}: R_1 \geq R_{\text{th}}(1) \end{aligned} \tag{11}$$

where R_1 is by (6) with $i = 1$. The rate threshold is analogous to the IT threshold of underlay CR systems [14]. Since the rate directly depends on SNR, the value of R_{th} can be chosen dynamically, i.e. in low-SNR region the noise power dominates so set R_{th} low in those regions and increase R_{th} in high-SNR regions.

To analyse the proposed AS method in terms of the rate performance of PU in IA-based CRN, we use outage probability metric. Outage probability is the probability that the given rate threshold cannot be satisfied because of channel variations. The outage probability of PU is

$$P_{\text{outage}-\text{PRX}} = \mathbb{P}\{\log_2(1 + \text{SINR}_{\max-\text{PRX}}) < R_{\text{th}}(1)\} \tag{12}$$

where $\text{SINR}_{\max-\text{PRX}}$ is the maximum SINR obtained at PRX after AS. The analytical outage probability expression for PU (considering interference from all SUs) is

$$\begin{aligned} &P_{\text{outage}-\text{PRX}} \\ &= \left[\prod_{j=2}^{K}\left[1 - \left(1 + \frac{(2^{R_{\text{th}}(1)} - 1)\sum_{j=2}^{K} P_j}{P_1}\right)^{-1} \mathrm{e}^{-\left(((2^{R_{\text{th}}(1)} - 1)\sigma_1^2)/P_1)\right)}\right]\right]^n \end{aligned} \tag{13}$$

Proof: See Appendix 1.

Since IA helps to cancel the interference, the outage probability expression without considering interference is

$$P_{\text{outage}-\text{PRX}} = \left[1 - \mathrm{e}^{-\left(((2^{R_{\text{th}}(1)} - 1)\sigma_1^2)/P_1)\right)}\right]^n \tag{14}$$

Also, the analytical expression for average BER [assuming binary phase shift keying (BPSK) modulation] for the proposed AS is

$$\begin{aligned} &\text{average BER}_{\text{AS}} = n \sum_{i=0}^{n-1} \binom{n-1}{i} (\\ &-1)^i \frac{1}{2(i+1)}\left[1 - \sqrt{\frac{P_1}{\sigma_1^2(i+1) + P_1}}\right] \end{aligned} \tag{15}$$

□

Proof: See Appendix 2.

In this work, we assume a central processor which has the channel information of the PU and all the SUs. Hence, to perform the IA-AS, the users get the global Channel State Information (CSI) from the central processor. The AS scheme considered in this paper selects the SURX antennas to improve the QoS of the PU subject to the constraint $R_{\text{th}}(1)$. Therefore, if none of the antenna combinations of SUs satisfy this threshold, then the SU is not permitted to access the PU's spectrum. Thus, an admission control is implemented for the SUs to access the PU spectrum moreover, utilising the benefits of AS.

Dynamic spectrum access or sharing can be achieved efficiently by establishing a spectrum market, where the PU leases its spectrum to SUs while protecting its QoS. Hence to optimise the process of spectrum allocation or sharing, resource management algorithms are utilised to enhance the QoS of the SUs. In this paper, we considered an adaptive PA algorithm by which the power is allocated to SUs in a centralised manner with an aim to guarantee the minimum rate requirement of each user also considering the proportional fairness among the users. □

3.2 Adaptive PA to improve QoS of PUs and SUs

The PA algorithms proposed in [17] allocate the minimum power required by the PU to it and allocate the remaining power to the SUs aiming at maximising the SU sum rates. Thus, QoS improvement of PUs is ignored here. In this paper, we propose an alternate PA method which aims at maximising the sum rate of all users, satisfying the minimum rate constraint, also considering the proportional fairness among the users. The proposed algorithm consists of three cases. We assume the total maximum power available as $P_{\max}$. The minimum power required for each user to satisfy their rate threshold is given by (9) for PU and (10) for SUs.

Case 1: $P_{\max} \leq P_{1-\min}$This implies that the total power available is less than the minimum power required by PU. Therefore, the rate requirement of PU cannot be met and all the available powers are allocated to PU alone.

Case 2: $P_{\max} \geq \sum_{i=1}^{K} P_{i-\min}$In this case, the rate thresholds of all users are satisfied and the optimum PA is performed to maximise the sum rate of all users. Thus, the problem statement is formulated as an optimisation problem

$$(\mathbf{P2}) \quad \max_{P_1 \ldots P_K} \sum_{i=1}^{K} \log_2\left(1 + \frac{P_i}{\sigma_i^2}|h_i|^2\right)$$

$$\text{such that, } P_i \geq P_{i-\min}, \ \forall i \qquad (16)$$

$$\sum_{i=1}^{K} P_i \leq P_{\max}$$

Assume $P_i = \hat{P}_i + P_{i-\min}$, the problem statement (P2) is reformulated as

Proof: See Appendix 3

$$(\mathbf{P3}) \quad \max_{\hat{P}_1 \ldots \hat{P}_K} \sum_{i=1}^{K} \log_2\left(1 + \frac{\hat{P}_i}{\sigma_i^2}|\hat{h}_i|^2\right)$$

$$\text{such that, } \hat{P}_i \geq 0, \ \ \forall i \qquad (17)$$

$$\sum_{i=1}^{K} (\hat{P}_i + P_{i-\min}) \leq P_{\max}$$

where $|\hat{h}_i|^2 = ((|h_i|^2\sigma_i^2)/(\sigma_i^2 + |h_i|^2 P_{i-\min}))$. The above optimisation problem is solved by the Lagrangian method and the closed-form expression for $\hat{P}_i$ is

$$\hat{P}_i = \left(\mu - \frac{\sigma_i^2}{|\hat{h}_i|^2}\right)^+ \qquad (18)$$

where μ is the Lagrange variable and it should satisfy

$$\mu = \frac{1}{K}\left[P_{\max} + \sum_{i=1}^{K}\left(\frac{\sigma_i^2}{|\hat{h}_i|^2} - P_{i-\min}\right)\right] \qquad (19)$$

Proof: See Appendix 3. Thus, the actual power allocated to all users, $i = 1, \ldots, K$ is $P_i = \hat{P}_i + P_{i-\min}$. This results in guaranteeing the minimum power for all users moreover, satisfying their minimum rate requirements. □

Case 3: $P_{1-\min} < P_{\max} < \sum_{i=1}^{K} P_{i-\min}$In this case, the minimum power required by PU is less than the maximum available power, but the sum of minimum powers of SUs is greater than maximum available power. Then, the intelligent way to solve this problem is to allocate PU with the minimum power required. Therefore, $P_1 = P_{1-\min}$ and for SUs the remaining power ($P_{\max} - P_{1-\min}$) is allocated to maximise their throughput and also increase their satisfaction toward their minimum rate requirements. We proposed a proportional fair PA for this case, i.e. maximising the geometric mean of the data rate of all users [18]. It helps to avoid having any user with very low data rate. The problem statement is formulated as

$$(\mathbf{P4}) \quad \max_{P_2 \ldots P_K} \sum_{i=2}^{K} \ln\left(\log_2\left(1 + \frac{P_i}{\sigma_i^2}|h_i|^2\right)\right)$$

$$\text{such that, } 0 < P_i \leq P_{i-\min}, \ \forall i \in \{2, \ldots, K\} \qquad (20)$$

$$\sum_{i=2}^{K} P_i \leq P_{\max} - P_{1-\min}$$

The above problem statement (P4) is a convex optimisation problem. Owing to ln(log(.)) operation in the objective function, it is difficult to derive the solution for optimum power P_i allocated to all SUs in terms of the Lagrange multipliers. We solve the problem using Convex optimization tool (CVX).

To analyse the performance of PA of SUs, Jain's index is used. It is a well-known measure of fairness [15]. The Jain's index FI , for a vector of SU rates $\boldsymbol{r}_{\text{SU}}$ is given by

$$\text{FI} = \frac{\left(\sum_{i=1}^{K-1} \boldsymbol{r}_{\text{SU}}(i)\right)^2}{(K-1)\sum_{i=1}^{K-1} \boldsymbol{r}_{\text{SU}}(i)^2} \qquad (21)$$

where $K-1$ is the total number of SUs, $\boldsymbol{r}_{\text{SU}} = [R_2, R_3, \ldots, R_K]^{\text{T}}$. The index FI takes the value of '1' when there is a complete fair allocation. The outage probabilities of SUs are similar to that defined in (14), and for the *i*th SURX, it can be written as

$$P_{\text{outage}-i\,\text{SRX}} = \left[1 - \mathrm{e}^{-\left(\left(2^{R_{\text{th}}(i)} - 1\right)\sigma_i^2/P_i\right)}\right]^n \qquad (22)$$

where $R_{\text{th}}(i)$ is the threshold rate for the *i*th SU, P_i is power allocated for the *i*th SU after performing adaptive PA and n indicates that the adaptive PA is done after AS.

4 Simulation results and discussion

We consider an IA-based underlay MIMO CRN which consists of 1 PU and 2 SUs ($K = 3$). The number of antennas at the TX for all users is assumed to be $M_i = 3, \ \forall\, i \in \{1, \ldots, K\}$. The PU RX has $N_1 = 3$ antennas, and all the SURXs has $L_i = 4, \ \forall\, i \in \{2, \ldots, K\}$ antennas. The number of data streams transmitted by each user d_i is assumed to be 1. The channel coefficients are assumed to be flat Rayleigh block fading. Also, it is assumed that perfect CSI is available at all nodes. The noise power σ_i^2 at each RX is normalised to unity. All the results are obtained by averaging over 1000 channel realisations.

First, we compare the performance improvement of PU in terms of its achievable rate (bps/Hz) by employing AS with IA. We choose MaxSINR and PIA-nSU IA algorithms. From $L_i = 4, \ \forall\, i \in \{2, \ldots, K\}$ antennas at SURXs, we choose $N_i = 3, \ \forall\, i \in \{2, \ldots, K\}$ receive antennas. Thus, totally there are 16 antenna combinations. From these one is selected to maximise the received SINR at PURX. It is assumed that equal power is allotted to all users. Since the noise power is unity, the power P_i indicates the SNR in the network. In Fig. 3, we compare MaxSINR IA algorithm with AS (MaxSINR-AS) and PIA-nSU IA with AS (PIA-nSU-AS) with the conventional IA algorithm (i.e. MaxSINR and PIA-nSU without AS). For conventional IA, we assume a situation with one random antenna combination. Also, the results are compared with the upper rate bound [15], i.e. the rate corresponds to maximum SINR of an MIMO system with no interference and it is given by

$$\text{Rate}_{\text{upper}} = \log_2(1 + \text{SINR}_{\max}) = \log_2\left(1 + \frac{P_1}{\sigma_1^2}\lambda_{\max}\left(\boldsymbol{H}_{11}^{\dagger}\boldsymbol{H}_{11}\right)\right) \qquad (23)$$

$$\lambda_{\max}\left(\boldsymbol{H}_{11}^{\dagger}\boldsymbol{H}_{11}\right) = \|\boldsymbol{H}_{11}\|_2^2 \qquad (24)$$

From Fig. 3, it can be seen that due to AS the transmission rates increases at high SNR. Comparing MaxSINR-AS and PIA-nSU-AS, at low SNR (0–5 dB) the performance of both are same as that of conventional, because the noise power dominates and it is difficult to achieve $R_{\text{th}}(1)$. For SNR > 5 dB the rates increase, but keep below Rate_upper. For low SNRs (0–13 dB) MaxSINR with AS is performing better. The performance of both algorithms is same during moderate SNRs (13–25 dB) and after that the rate of PIA-nSU with AS increases and it remains close to the upper bound. Fig. 4 shows the comparison of outage probability of PU versus SNR of MaxSINR-AS, PIA-nSU-A,S and (13), (14), with $R_{\text{th}}(1) = 5$ bps/Hz and $R_{\text{th}}(1) = 2.5$ bps/Hz. The outage probability of MaxSINR-AS algorithm is less than PIA-nSU-AS algorithm, because at low SNRs MaxSINR algorithm performs better. The outage probabilities obtained by both algorithms are less than the theoretical bound. Fig. 5 shows the improved performance of PU in terms of average BER due to AS.

After performing AS at SURXs, we perform fairness-based adaptive PA to satisfy the minimum rate requirements of PUs as

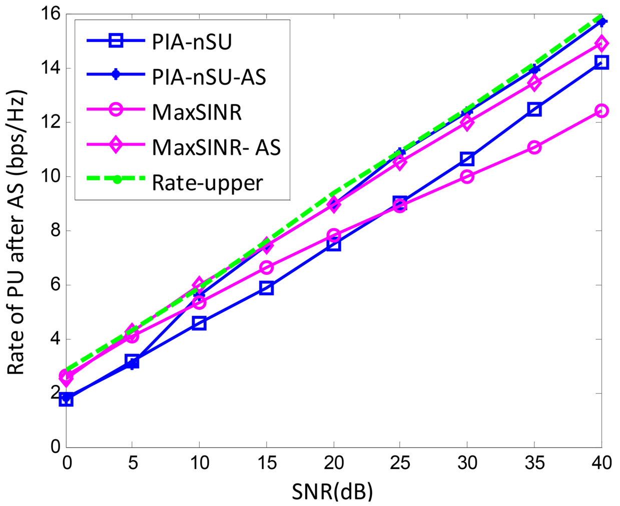

Fig. 3 *PU rate versus SNR by MaxSINR, PIA-nSU, MaxSINR-AS, PIA-nSU-AS and Rate_upper for MIMO system, $R_{th}(1) = 5\,bps/Hz$*

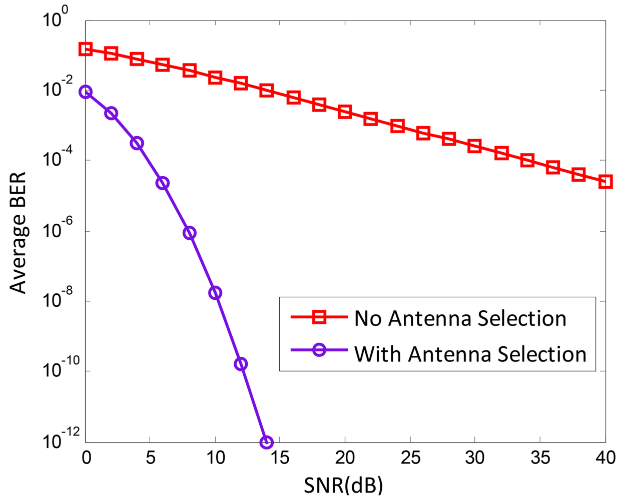

Fig. 5 *Average BER of PU versus SNR with AS (15) and without AS, assuming BPSK modulation, $R_{\rm th}(1) = 5\,\rm bps/Hz$*

well as SUs and also to improve the QoS of both users. We proceeded with the antenna combination selected during AS procedure. In all the previous graphs, it is assumed that equal power is allocated for all users and is equal to the SNR with the assumption that the noise power is equal to unity. Therefore, the total maximum power available for users before PA is $P_{\max} = \text{SNR} \times K$. Fig. 6 shows the comparison of the performance of MaxSINR, PIA-nSU IA with and without the proposed adaptive PA. It can be seen that the sum rate of SUs for both algorithms with PA (MaxSINR-PA and PIA-nSU-PA) closely follow the corresponding without PA. At low SNR (0–10 dB), MaxSINR-PA shows a small increase in sum rate and at high SNRs it closely follows the MaxSINR with equal PA. Considering the PIA-nSU-PA algorithm, its sum rate remains same as that of with equal PA at moderate SNR, but at low and high SNRs it closely follows the other. This is because the proposed PA aims at improving the fairness, so that the minimum required rate is satisfied. The rate threshold for all users assumed for this graph is 5 bps/Hz.

The performance of the proposed adaptive PA for each SU in terms of outage probability is shown in Fig. 7. We assume the minimum rate requirement for all users in this graph as $5\,\rm bps/Hz$. The outage probability achieved through MaxSINR-PA for both users is less than that of PIA-nSU-PA algorithm. This is because at low SNRs the rate achieved through MaxSINR algorithm is larger than that of PIA-nSU algorithm. Fig. 8 shows the Jain's fairness index achieved for SUs by employing the proposed adaptive PA for different rate requirements ($R_{\rm th}$). From this figure, we can study that when the rate requirements of both the SUs are same, then the power is allocated in such a way that both users will get almost equal shares, thus improving fairness. When the minimum rate

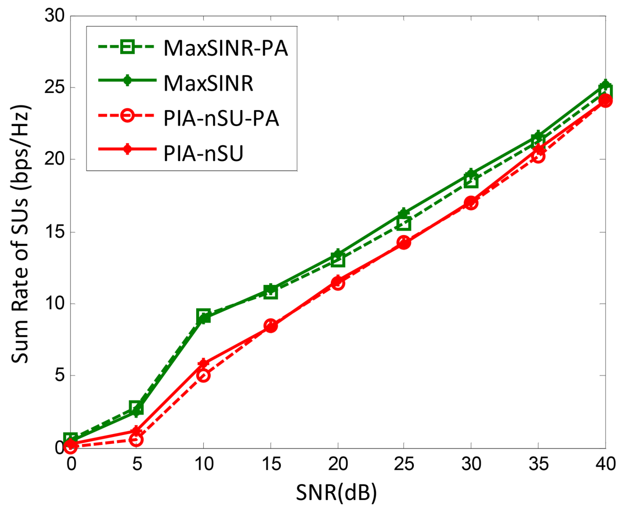

Fig. 6 *SU sum rate versus SNR by MaxSINR-PA, MaxSINR, PIA-nSU-PA and PIA-nSU $R_{th}(1) = 5\,bps/Hz$, $R_{th}(2) = 5\,bps/Hz$, $R_{th}(3) = 5\,bps/Hz$*

requirements differ for each SU, then the power is allocated in such a way that it tries to satisfy each users rate requirements and also the fairness. This will be more clearly understood from Table 1.

Table 1 presents the comparison of achievable rates against the rate requirements of each SU. The results for SNR of 5 and 10 dB are presented. First, consider the case with SNR = 5 dB, $R_{\rm th}(2) = 1\,\rm bps/Hz$, $R_{\rm th}(3) = 5\,\rm bps/Hz$ under MaxSINR algorithm. At low SNRs, the actual sum rate achievable will be low. Also,

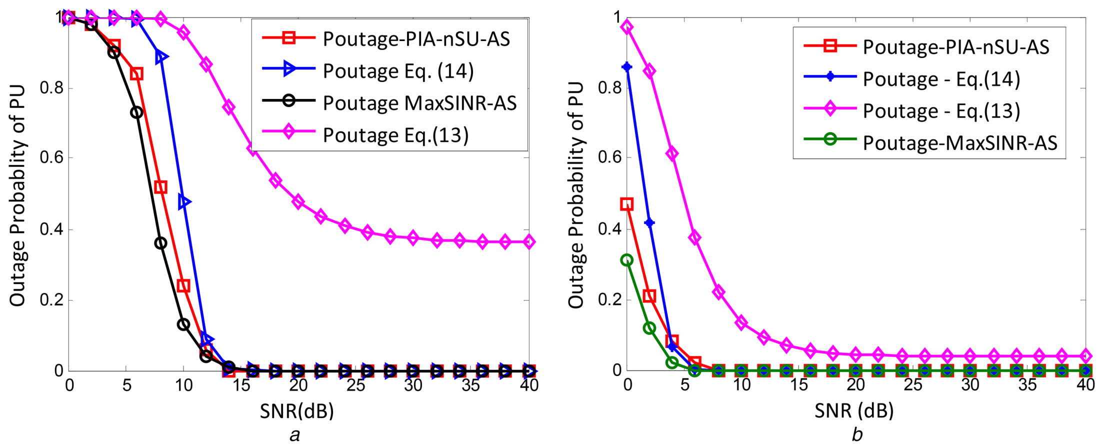

Fig. 4 *Outage probability of PU versus SNR by MaxSINR-AS, PIA-nSU-AS, (13) and (14)*
(a) *$R_{\rm th}(1) = 5\,\rm bps/Hz$*, ***(b)*** *$R_{\rm th}(1) = 2.5\,\rm bps/Hz$*

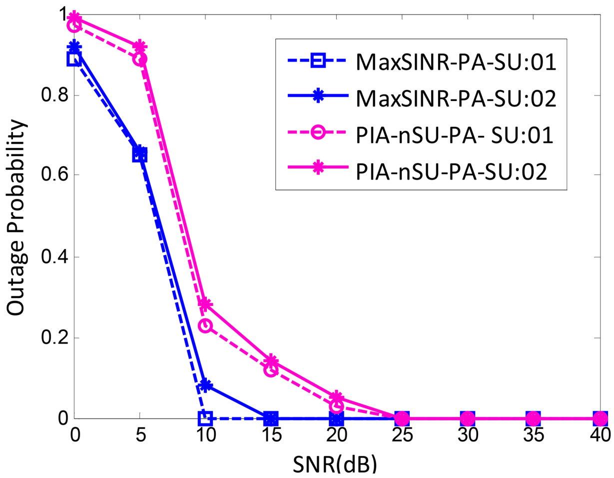

Fig. 7 *Outage probability of each SU versus SNR by MaxSINR-PA, PIA-nSU-PA, $R_{th}(1) = 5\,bps/Hz$, $R_{th}(2) = 5\,bps/Hz$, $R_{th}(3) = 5\,bps/Hz$*

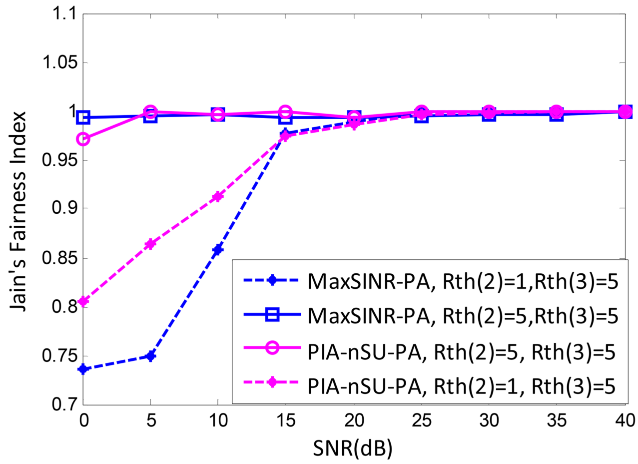

Fig. 8 *Jain's fairness index of SUs rates versus SNR by MaxSINR-PA and PIA-nSU-PA with $R_{th}(2) = 5\,bps/Hz$, $R_{th}(3) = 5\,bps/Hz$ and $R_{th}(2) = 1\,bps/Hz$, $R_{th}(3) = 5\,bps/Hz$*

Table 1 Achievable rate and $R_{\rm th}$ of each SU

		MaxSINR-PA algorithm		
SNR	Minimum rate $R_{\rm th}(2)$, bps/Hz	Rate achieved User:01	Minimum rate $R_{\rm th}(3)$, bps/Hz	Rate achieved User:02
5 dB	1.0000	0.6273	5.0000	2.3564
	5.0000	1.4737	5.0000	1.2813
10 dB	1.0000	2.3869	5.0000	5.6525
	5.0000	4.9446	5.0000	4.3407
		PIA-nSU-PA algorithm		
5 dB	1.0000	0.2834	5.0000	0.6465
	5.0000	0.4008	5.0000	0.4705
15 dB	1.0000	2.7824	5.0000	4.5770
	5.0000	3.2911	5.0000	3.7332

Table 2 Complexity of various algorithms

Algorithm	Complexity	Required number of flops
MinIL	$9K(N^3+M^3)+K(K-1)[4NMd+2d(N^2+M^2)-d(N+M)]$	1.854×10^3
MaxSINR	$Kd(N^3+M^3)\frac{2}{3}+(N^2+M^2)(3+K(1+d))+(N+M)(1-kd)+4KdMN-2$	0.876×10^3
PIASU	$(K-1)\left[6NdM+7N^2-d(M+N-1)+2M^2d+\frac{2}{3}N^3d+9M^3+d^2K\left[2NM+\frac{N^2}{d}+N^2-N\right]\right]$ $+9KN^3+4M^2d+8Md^2+9d^3+(K-2)[4NdM-d(M+N)+2d(N^2+M^2)]$	1.844×10^3
PIA-nSU	$(K-1)\left[Md(2N-1)+2d(N^2+M^2)+9M^3+N^2(3d+Kd+d^2K)+d^2K(2MN-N)+\frac{2}{3}N^3d-d\right]$ $+9N^3+4M^2d+8Md^2+9d^3$	1.186×10^3
AS	$(K-1)n[(2N-1)(2Nd+2d^2)+N+N^2]\times$ complexity_IA	$1.664\times10^3\times complexity\_IA$
adaptive PA	for case 1: the complexity is negligibly small for case 2: the complexity is $\mathcal{O}(K^2)$ for case 3: we solved using CVX tool	

since the rate requirements differ, the proposed method manages the PA in such a way that it tries to satisfy the minimum rate requirement and also the proportional fairness. Therefore, the rate achieved by user-1 is 0.6273 bps/Hz and user-2 is 2.3564 bps/Hz. While for the case of 10 dB (the actual sum rate achievable will be sufficient to meet the rate requirements), it can be seen from this table that the rate requirements are satisfied and it will look for maximising the sum rate. Therefore, the rate achieved by user-1 is 2.3869 bps/Hz [against the minimum rate $R_{\rm th}(2) = 1$ bps/Hz] and user-2 is 5.6525 bps/Hz [$R_{\rm th}(3) = 5$ bps/Hz]. The explanation for the remaining cases goes in the same manner as above.

4.1 Note on complexity

We present the complexity of different algorithms studied in this paper by counting the number of floating point operations (flops) in Table 2. For the sake of brevity, we omit the detailed evaluation of complexity. The required number of flops by each algorithm for $K = 3$, $M_i = 3$, $\forall i \in \{1, \ldots, K\}$, $N_i = 3$, $\forall i \in \{1, \ldots, K\}$, d_i is $= 1$, $\forall i \in \{1, \ldots, K\}$
provided in this table. While comparing the complexities of different IA schemes, MaxSINR assumes the least complexity, followed by PIA-nSU. The complexity of the AS algorithm and the proposed adaptive PA algorithm is also presented.

5 Conclusion

In this paper, we proposed an efficient method to improve the QoS of PUs and SUs in an underlay CR scenario. We first compared the performances of different IA algorithms for underlay CRN which eliminates the interference caused by the SUs to the PUs also the interference at the SUs due to other users. Next, we proposed an IA-based AS strategy to improve the performance of PUs. We presented the outage probability analysis and BER analysis for the proposed AS strategy. Furthermore, when the PUs involved in spectrum leasing, in order to guarantee certain QoS for SUs, we propose an adaptive PA strategy for IA-based CRN, which aims at allocating power based on fairness to all users as well as guaranteeing the minimum rate requirement for all users. Simulation results presented shows that there is a significant improvement in QoS for all users in terms of SNR with the proposed method. We also presented the fairness analysis of the proposed adaptive PA in terms of Jain's fairness index.

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7 Appendix

7.1 Appendix 1

7.1.1 Proof of (13): Using AS we choose a set of antennas at SURX, so that SINR at the PURX is maximised. If there are n different possible antenna subsets of SURXs, then $\mathrm{SINR}_{\max}$ at PURX is

$$\mathrm{SINR}_{\max-\mathrm{PRX}} = \max_{i=1,\dots,n} \{\mathrm{SINR}_{i-\mathrm{PURX}}\} \tag{25}$$

where $\mathrm{SINR}_{i-\mathrm{PURX}}$ corresponds to the SINR at PURX due to the ith antenna combination. The CDF of $\mathrm{SINR}_{\max-\mathrm{PRX}}$ is $F_{\mathrm{SINR}_{\max-\mathrm{PRX}}}(\gamma) = \left[F_{\mathrm{SINR}_{i-\mathrm{PRX}}}(\gamma)\right]^n$. The instantaneous SINR at the PURX due to the ith antenna subset is

$$\mathrm{SINR}_{i-\mathrm{PURX}} = \frac{P_1|h_1|^2}{\sum_{j=2}^{K} P_j|g_j|^2 + \sigma_1^2} \tag{26}$$

where P_1 is the TX power of PU, P_j is the TX power of SUs, $|h_1|^2$ is the channel gain of PU after AS, $|g_j|^2$ is the interference channel gain from SUTX to PURX for the ith antenna subset. $|h_1|^2$, $|g_j|^2$ follows exponential distribution with unit mean and variance. The CDF of instantaneous SINR at the PURX due to the ith antenna subset is obtained as

$$\begin{aligned} F_{\mathrm{SINR}_{i-\mathrm{PURX}}}(\gamma) &= \mathbb{P}(\mathrm{SINR}_{i-\mathrm{PURX}} \le \gamma) \\ &= \mathbb{P}\left(|h_1|^2 \le \frac{\gamma}{P_1}\left(\sum_{j=2}^{K} P_j|g_j|^2 + \sigma_1^2\right)\right) \end{aligned} \tag{27}$$

Let $X = |h_1|^2$, $Y_j = |g_j|^2$, $\forall j = 2, \dots, K$. Therefore, $F_{\mathrm{SINR}_{i-\mathrm{PURX}}}(\gamma)$ equation (27) can be written as,

$$\begin{aligned} &\int_0^{\infty} \cdots \int_0^{\infty} \left[\int_0^{(\gamma/P_1)\left(\sum_{j=2}^{K} P_j Y_j + \sigma_1^2\right)} f_X(x)\mathrm{d}x\right] \\ &\qquad \prod_{j=2}^{K} f_{Y_j}(y)\mathrm{d}y_2, \dots, \mathrm{d}y_K \\ &= \int_0^{\infty} \cdots \int_0^{\infty} \left[\int_0^{(\gamma/P_1)\left(\sum_{j=2}^{K} P_j Y_j + \sigma_1^2\right)} \mathrm{e}^{-x}\mathrm{d}x\right] \\ &\qquad \prod_{j=2}^{K} \mathrm{e}^{-y_j}\mathrm{d}y_2, \dots, \mathrm{d}y_K \\ &= \int_0^{\infty} \cdots \int_0^{\infty} \left[1 - \mathrm{e}^{-(\gamma/P_1)\sum_{j=2}^{K} P_j y_j} \mathrm{e}^{-(\gamma/P_1)\sigma_1^2}\right] \\ &\qquad \prod_{j=2}^{K} \mathrm{e}^{-y_j}\mathrm{d}y_2, \dots, \mathrm{d}y_K \\ &= \prod_{j=2}^{K}\left[\int_0^{\infty}\left[\mathrm{e}^{-y_j} - \mathrm{e}^{-\left((\gamma/P_1)\sum_{j=2}^{K} P_j + 1\right)y_j} \mathrm{e}^{-(\gamma/P_1)\sigma_1^2}\right]\right]\mathrm{d}y_j \\ &= \prod_{j=2}^{K}\left[1 - \left(1 + \frac{\gamma}{P_1}\sum_{j=2}^{K} P_j\right)^{-1} \mathrm{e}^{-(\gamma/P_1)\sigma_1^2}\right] \end{aligned} \tag{28}$$

If we neglect the interference, since IA eliminates the interference

$$F_{\mathrm{SINR}_{i-\mathrm{PURX}}}(\gamma) = \left[1 - \mathrm{e}^{-(\gamma/P_1)\sigma_1^2}\right] \tag{29}$$

The CDF of $\mathrm{SINR}_{\max-\mathrm{PRX}}$ is

$$\begin{aligned} F_{\mathrm{SINR}_{\max-\mathrm{PRX}}}(\gamma) &= \left[F_{\mathrm{SINR}_{i-\mathrm{PURX}}}(\gamma)\right]^n \\ &= \left[\prod_{j=2}^{K}\left[1 - \left(1 + \frac{\gamma}{P_1}\sum_{j=2}^{K} P_j\right)^{-1} \mathrm{e}^{-(\gamma/P_1)\sigma_1^2}\right]\right]^n \end{aligned} \tag{30}$$

Neglecting the interference term

$$F_{\mathrm{SINR}_{\max-\mathrm{PRX}}}(\gamma) = \left[1 - \mathrm{e}^{-\left((\gamma/P_1)\sigma_1^2\right)}\right]^n \tag{31}$$

The outage probability is

$$\begin{aligned} P_{\mathrm{outage-PRX}} &= \mathbb{P}\{\log_2(1 + \mathrm{SINR}_{\max-\mathrm{PRX}}) < R_{\mathrm{th}}(1)\} \\ &\mathbb{P}\left\{\mathrm{SINR}_{\max-\mathrm{PRX}} < 2^{R_{\mathrm{th}}(1)} - 1\right\} \\ &= F_{\mathrm{SINR}_{\max-\mathrm{PRX}}}\left(2^{R_{\mathrm{th}}(1)} - 1\right) \end{aligned} \tag{32}$$

$$= \left[\prod_{j=2}^{K}\left[1 - \left(1 + \frac{(2^{R_{\mathrm{th}}(1)} - 1)\sum_{j=2}^{K} P_j}{P_1}\right)^{-1} \mathrm{e}^{-\left(((2^{R_{\mathrm{th}}(1)} - 1)\sigma_1^2)/P_1\right)}\right]\right]^n \tag{33}$$

For IA-based CR system

$$P_{\mathrm{outage-PRX}} = \left[1 - \mathrm{e}^{-\left(((2^{R_{\mathrm{th}}(1)} - 1)\sigma_1^2)/P_1\right)}\right]^n \tag{34}$$

7.2 Appendix 2

7.2.1 Proof of (15): Average BER at PURX due to AS is

$$\begin{aligned} &\mathbb{E}\left[Q\left(\sqrt{2|h_1|^2\frac{P_1}{\sigma_1^2}}\right)\right] \\ &= \int_0^\infty Q\left(\sqrt{2|h_1|^2\frac{P_1}{\sigma_1^2}}\right) f_{\mathrm{SINR}_{\max-\mathrm{PRX}}}(\gamma)\mathrm{d}\gamma \end{aligned} \tag{35}$$

From the order statistics theory, the PDF of $\mathrm{SINR}_{\max-\mathrm{PRX}}$ is (see (36))

Therefore

$$\begin{aligned} f_{\mathrm{SINR}_{\max-\mathrm{PRX}}}(\gamma) &= n\frac{\sigma_1^2}{P_1}\mathrm{e}^{-(\gamma\sigma_1^2/P_1)}\left[1 - \mathrm{e}^{-(\gamma/P_1)\sigma_1^2}\right]^{n-1} \\ &= n\frac{\sigma_1^2}{P_1}\sum_{i=0}^{n-1}\binom{n-1}{i}(-1)^i \mathrm{e}^{-\gamma(i+1)(\sigma_1^2/P_1)} \end{aligned} \tag{37}$$

Substituting (37) into (35), average BER $= n(\sigma_1^2/P_1)\sum_{i=0}^{n-1}\binom{n-1}{i}(-1)^i\int_0^\infty Q(\sqrt{2\gamma})\mathrm{e}^{-\gamma(i+1)(\sigma_1^2/P_1)}\,\mathrm{d}\gamma$.

After evaluating the integral, the BER expression is

$$\text{average BER} = \frac{n}{2}\sum_{i=0}^{n-1}\binom{n-1}{i}\frac{(-1)^i}{i+1}\left[1 - \sqrt{\frac{P_1}{\sigma_1^2(i+1) + P_1}}\right] \tag{38}$$

7.3 Appendix 3

7.3.1 Proof of problem statement P3 and (18), (19): Assume

$$P_i = \hat{P}_i + P_{i-\min} \tag{39}$$

Therefore, the objective function in P2 is rewritten as

$$\begin{aligned} &\max_{P_1 \ldots P_K}\sum_{i=1}^{K}\log_2\left(1 + \frac{(\widehat{P_i + P_{i-\min}})}{\sigma_i^2}|h_i|^2\right) \\ &= \sum_{i=1}^{K}\log_2\left(1 + \frac{\hat{P}_i}{\sigma_i^2}|h_i|^2 + \frac{P_{i-\min}}{\sigma_i^2}|h_i|^2\right) \\ &= \sum_{i=1}^{K}\log_2\left(\left(1 + \frac{P_{i-\min}}{\sigma_i^2}|h_i|^2\right)\left(1 + \frac{\hat{P}_i|h_i|^2/\sigma_i^2}{\frac{\sigma_i^2 + P_{i-\min}|h_i|^2}{\sigma_i^2}}\right)\right) \\ &= \sum_{i=1}^{K}\log_2\left(\left(1 + \frac{P_{i-\min}}{\sigma_i^2}|h_i|^2\right)\left(1 + \frac{\hat{P}_i|\hat{h}_i|^2}{\sigma_i^2}\right)\right) \\ &= \sum_{i=1}^{K}\log_2\left(\text{constant} + \left(1 + \frac{\hat{P}_i|\hat{h}_i|^2}{\sigma_i^2}\right)\right) \end{aligned} \tag{40}$$

where

$$|\hat{h}_i|^2 = \frac{\sigma_i^2|h_i|^2}{\sigma_i^2 + P_{i-\min}|h_i|^2} \tag{41}$$

Thus the problem statement P2 is rewritten as P3. The Lagrangian for P3 can be written as

$$J(\hat{P}_1, \ldots, \hat{P}_K) = -\sum_{i=1}^{K}\log_2\left(1 + \frac{\hat{P}_i|\hat{h}_i|^2}{\sigma_i^2}\right) + \lambda\sum_{i=1}^{K}\left(\hat{P}_i + P_{i-\min}\right) - P_{\max} \tag{42}$$

where λ is the Lagrangian multiplier

$$\frac{\partial J}{\partial \hat{P}_i} = \frac{-1}{1 + (\hat{P}_i|\hat{h}_i|^2/\sigma_i^2)}\frac{|\hat{h}_i|^2}{\sigma_i^2} + \lambda = 0$$

Solving the above equation

$$\hat{P}_i = \left(\frac{1}{\lambda} - \frac{\sigma_i^2}{|\hat{h}_i|^2}\right)^+ \tag{43}$$

where $(1/\lambda) = \mu$, and μ should satisfy

$$\begin{aligned} &\sum_{i=1}^{K}\left(\left(\mu - \frac{\sigma_i^2}{|\hat{h}_i|^2}\right)^+ + P_{i-\min}\right) = P_{\max} \\ &\Rightarrow \mu = \frac{1}{K}\left[P_{\max} + \sum_{i=1}^{K}\left(\frac{\sigma_i^2}{|\hat{h}_i|^2} - P_{i-\min}\right)\right] \end{aligned} \tag{44}$$

$$\begin{aligned} f_{\mathrm{SINR}_{\max-\mathrm{PRX}}}(\gamma) &= \frac{\mathrm{d}}{\mathrm{d}\gamma}F_{\mathrm{SINR}_{\max-\mathrm{PRX}}}(\gamma) = \frac{\mathrm{d}}{\mathrm{d}\gamma}\left[F_{\mathrm{SINR}_{i-\mathrm{PURX}}}(\gamma)\right]^n \\ &= n\left[F_{\mathrm{SINR}_{i-\mathrm{PRX}}}(\gamma)\right]^{n-1}\frac{\mathrm{d}}{\mathrm{d}\gamma}F_{\mathrm{SINR}_{i-\mathrm{PRX}}}(\gamma) \\ &= nf_{\mathrm{SINR}_{i-\mathrm{PRX}}}(\gamma)\left[F_{\mathrm{SINR}_{i-\mathrm{PRX}}}(\gamma)\right]^{n-1} \\ f_{\mathrm{SINR}_{i-\mathrm{PRX}}}(\gamma) &= \frac{\mathrm{d}}{\mathrm{d}\gamma}F_{\mathrm{SINR}_{i-\mathrm{PRX}}}(\gamma) = \frac{\mathrm{d}}{\mathrm{d}\gamma}\left[1 - \mathrm{e}^{-(\gamma/P_1)\sigma_1^2}\right] = \frac{\sigma_1^2}{P_1}\mathrm{e}^{-(\gamma\sigma_1^2/P_1)} \end{aligned} \tag{36}$$