



OPEN Shape-dependent thermal transport in bioconvective dissipative flow of hybrid nanofluid with Cattaneo–Christov heat flux

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Thermal performance of lubricant-based hybrid nanomaterial has attracted significant attention due to their potential for advanced heat transfer applications. However, the combined effects of nanoparticle shape, dissipation features, thermal radiation, bioconvection, heat source and non-Fourier's heat flux model have not been studied within a unified framework. This investigation analyses the bioconvective dissipative flow of a lubricant-based hybrid nanofluid confined by a porous stretching surface. The hybrid nanofluid is prepared by dispersing zinc oxide (ZnO) and molybdenum disulphide (MoS₂) nanoparticles in SAE10W-30 engine oil. The mathematical model accounts the nonlinear radiated effects, internal heat source, viscous dissipation, and motile microorganisms, while heat transport is characterized using the Cattaneo–Christov heat flux model. The transformed system of equations has been solved numerically with help of shooting scheme. The assessment of five nanoparticle shapes (sphere, platelet, blade, brick, and cylinder) on the flow, thermal, concentration, and microorganism distributions are focused for both mono and hybrid nanofluids. The results claim that thermal radiation and internal heat generation significantly improve the thermal profile, whereas viscous dissipation further elevates fluid temperature due to frictional heating. The blade-shaped nanoparticles provide the highest thermal enhancement among the considered geometries. These findings demonstrate the important role of nanoparticle geometry and hybridization in improving thermal transport characteristics under non-Fourier heat conduction.

Keywords Hybrid nanofluid, Bioconvective flow, Cattaneo–Christov heat flux, Effects, Shape features, Numerical simulations.

List of symbols

(u, v)	Velocity components (ms^{-1})
(x, y)	Coordinate axis (m)
T	Temperature (K)
C	Concentration (kgm^{-3})
ρ_{hnf}	Density (kgm^{-3})
B_0	Magnetic field intensity ($\text{kgs}^{-2}\text{A}^{-1}$)
σ_{hnf}	Electrical conductivity (Sm^{-1})
C_∞	Ambient concentration (kgm^{-3})
σ^*	Stefan Boltzmann constant ($\text{Wm}^{-2}\text{K}^{-4}$)
C_w	Surface concentration (kgm^{-3})
k_{hnf}	Thermal conductivity

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$(\rho C_p)_{hnf}$	Specific the HNF heat capacity
n	Microorganism's density
h_f	Heat transfer coefficient
Q_0	Heat source
D_B	Brownian motion diffusivity
T_∞	Ambient temperature
w_m	Swimming cells speed
D_m	Microorganisms diffusion
b_m	Chemotaxis constant.
S	Magnetic parameter
P_0	Porosity number
δ_E	Thermal relaxation number
Pr	Prandtl number
R	Radiation number
Γ	Suction parameter
Bi	Biot number
Sc	Schmidt number
δ_C	Thermal concentration number
Lb	Bio-convective Lewis number
Pe	Peclet number
σ_m	Motile difference constant
λ_Q	External heat source
Ec	Eckert number
T_f	Convective temperature
n_w	Surface microorganism's density
f'	Dimensionless velocity profile
θ	Dimensionless temperature
ϕ	Dimensionless concentration
χ	Dimensionless microorganism's density
Re_x	Reynolds number
Sh	Sherwood number
Nu	Nusselt number
C_f	Skin friction coefficient

A significant attention towards the thermal prospective of hybrid nanofluid is intended due to their impressive ability to improve thermal performance in industrial systems and engineering phenomena. Amongst different categories of nanomaterials, hybrid nanofluids (HNF) are emerged as a promising class modern thermal fluids. The HNF are prepared with suspension of two or more different categories of nanoparticles in base liquid, resulting stable and enhanced thermo-physical consequences as compared to mono nanofluids. The synergistic interaction amongst various nanoparticles improve the stability, thermal conductivity and energy transport features, making the HNF suitable for applications like cooling of automobiles, renewable energy systems, heat exchangers, nuclear systems etc. A diverse attention is paid to investigation of nanofluid flow in different configurations. Pandey et al.¹ described the thermal attention of nanofluid in cone and disk flow by examining the different nanoparticles aggregation. Oudina et al.² analyzed the thermal relevance due to nanofluid in porous media under the entropy generation impact. Bhatti and Ellahi et al.³ focused to non-Darcian insight into thermal analysis of nanofluid across an elastic surface by endorsing the thermal and velocity slip managements. The hybrid nanofluid analysis in porous regime for peristaltic flow has been intended by Hayat et al.⁴. Sandeep et al.⁵ discussed the Brownian movement in Carreau and Casson hybrid nanofluid due to moving wedge. Thermal consequences of tri hybrid nanofluid with biomedical therapeutics has been responded by Salih et al.⁶. Sarkar et al.⁷ addressed the unsteady flow of Casson HNF with quadratic thermal radiated effects to improve the energy management in heat transfer systems. The sensitivity analysis due to HNF over a radiated surface was explained by Ontela et al.⁸. Tlili⁹ addressed the heat generation effects to explore enhancement of thermal features of HNF. Alam et al.¹⁰ analyzed the tri hybrid nanofluid transport due to porous medium via numerical simulations. Farooq et al.¹¹ observed neural network simulations to analyse the squeezing flow of hybrid nanofluid. Discussion of ternary Eyring-Powell hybrid nanofluid with bioconvection assessment was focused by Farooq et al.¹². Maddina et al.¹³ described the hybrid nanofluid properties in a rectangular fins with applications of sustainable energy systems. Wang et al.¹⁴ intended the thin film flow of ternary hybrid nanofluid comprising the accelerating boundary constraints. Ratha et al.¹⁶ expressed the applications of ternary HNF in sensor devices. An unsteady squeezing flow of THNF due to parallel surface was claimed by Ratha et al.¹⁶. Qamar et al.¹⁷ utilized the slip influence to examine the heat transfer with irregular trend associated to hybrid nanofluid. Qamar et al.¹⁸ expressed the magnetic dipole interference in analysing the heat transfer due to hybrid nanofluid. Said et al.¹⁹ used a non-classical Fourier model to account the heat transfer simulations in modified rate type hybrid nanofluid. Mursaleen et al.²⁰ investigated the Darcy medium applications in hybrid nanofluid flow and trained the model with help of machine learning algorithm. Kanti et al.²¹ presented the observations for solar collectors efficiency of hybrid nanofluid via experimental modelling.

An important factor impacting the thermal prospective of nanomaterials is the shape of nanoparticles. The shape features attribute novel importance in inspecting the effective surface area and improvement of thermal conductivity. Distinct configurations like platelet, cylindrical, spherical and blade nanoparticles explore diverse

thermal behavior, leading to fluctuation in heat transfer rates and flow features. Based on available studies, it has been noticed that non-spherical nanoparticles present peak thermal conductivity due to higher surface to volume ratio and enhanced Brownian diffusion. Therefore, accounting the shape dependent inspection into HNF is important to accurate prediction and optimization of thermal transport systems. Hussain et al.²² examined the particles shape features in EMHD flow of nanofluid. The Newtonian heating and shape feature analysis for alloy based nanofluid was determined by Muhammad et al.²³. Sreelakshmi et al.²⁴ utilized the thermal simulations for five different shape features in thin film flow of hybrid nanofluid. The analysis for shape features across a slender surface was visualized by Sandeep et al.²⁵. Hajlaoui et al.²⁶ identified the importance of shape factors in study of HNF under the viscous dissipation and chemical reaction effects. Jat et al.²⁷ analyzed the nonlinear stretching surface flow of hybrid nanomaterials by involving the shape features. Ghachem et al.²⁸ investigation the hybrid nanofluid analysis with understanding of different shape features and finite propagation effects.

In addition to enhancement of thermal dynamics, the bioconvection has attained importance in nanofluid research. The phenomenon of bioconvection is based on collective movement of self-propelled microbe's suspension in fluid, inducing density gradient and impacted the fluid movement. The bioconvection successfully impact the flow stability, mixing features and heat transfer performance. When microorganisms are incorporated into nanomaterials, they result an additional transport dynamic that increases the suspension stability of nanoparticles and enhances overall energy transport. This phenomenon is relevant in biomedical engineering, bio-thermal systems and advanced cooling technologies where enhanced and controlled mixing is necessitated. Shah et al.²⁹ addressed the suspension of microorganisms in hybrid nanofluid due to starched surface. Hussain et al.³⁰ analyzed the significance of oxytactic bacteria in a lid driven cavity filled with hybrid nanofluid. Alshammari et al.³¹ predicted the thermal non- equilibrium on assessment of bioconvection of hybrid nanofluid. The analysis was performed for both classical and modified Hamilton-Crosser models. The results for thermo-bioconvection in hybrid nanofluid filled with enclosure was revealed by Ahlawat et al.³². Hamida et al.³³ analyzed the transport of bioconvection in hybrid nanofluid with suspension of gyrotactic microbes.

Motivated by these considerations, current investigation presents improved energy transport in bioconvective magnetized flow of HNF consisting of MoS₂ and ZnO nanoparticles dispersed in SAE10W-30 engine oil over a porous elastic surface. The mathematical model is developed by including the applications of radiated effects, viscous dissipation and additional heat source. The Cattaneo–Christov heat flux model has been used to analyze the heat and mass transfer performance. The heat transfer phenomenon is further supported with choice of five different shape features like five different shape features like sphere, platelet, blade, brick and cylinder. For solution methodology, a famous shooting technique is implemented by ensuring the accuracy. A comparative analysis is presented for mono and hybrid nanofluids. The motivation of continuation stems from increasing demand of peak-performance lubricants capable of enhancing the heat dissipation and energy efficiency in the advanced thermal management systems. The lubricant based hybrid nanomaterial with suspension of ZnO and MoS₂ nanoparticles have intended the attention due to novel tribological features and impressive heat transfer capability. Simultaneously, understanding the impact of nanoparticle configuration and finite-speed heat propagation presented by non-Fourier's model is essential for design of modern lubrication and cooling technologies. Although bioconvection associated to microorganisms is primarily a theoretical transport phenomenon and does not imply the physical relevant of microorganisms in the engine-lubricants, it utilizes value insights into the coupled momentum, thermal, concentration and microorganism transport dynamics that are employed in mathematical fluid dynamics. Therefore, the proposed model aims to analyze combined prospective of nanoparticles shape, radiated effects, viscous dissipation, internal heat transfer and non-Fourier's conduction on transport of ZnO-MoS₂/ SAE10W-30 hybrid nanofluids, providing a theoretical framework for the development of advanced thermal management and lubrication systems.

This investigation aims to find the answer to following research questions:

- How do the hybrid nanofluid impact the thermal, concentration and microorganism transport features as compared to mono nanomaterial?
- What is the impact of nanoparticle configuration on analyzing thermal transport of ZnO-MoS₂/SAE10W-30 hybrid nanofluid?
- How do the updated Fourier's model, dissipation effects, radiated phenomenon, porous medium and heat generation collectively fluctuates the transport behavior of the hybrid nanofluid?

Mathematical modelling of problem

Following assumption are followed to develop the governing equations:

- A two-dimensional, steady-state and laminar flow of HNF has been considered over elastic porous stretched surface as shown in Fig. 1.
- The driven flow due to elastic surface is subject due to uniform velocity $u = u_\omega = c_0 x$ with stretching constant c_0 while x be coordinate axis.
- Symbolizing the temperature with T , concentration with C while n is used to express the microorganisms density.
- In energy equation, the convective terms present the transport of heat by moving fluid, where thermal diffusion term presents the heat conduction through effective thermal conductivity of hybrid nanomaterial.
- The viscous dissipation effects are accounted for illustrating the convection of mechanical energy into thermal energy due to friction.
- The radiated phenomenon with nonlinear expression are considered to enhance the thermal transport features.

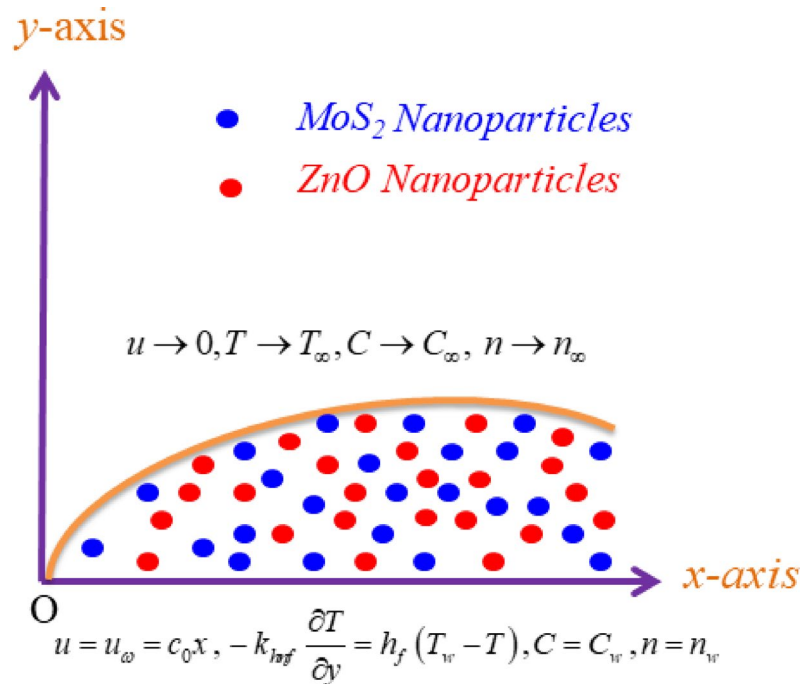


Fig. 1. Flow geometry of the problem.

- The relations of advanced Fourier's model are followed in concentration and heat equations to present the thermal propagation features.
- The magnetic force implementation is normally assumed.

The continuity equation is expressed in Eq. (1).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

Equation (2) presents the momentum equation¹⁹.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{hnf} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{hnf} B_0^2}{\rho_{hnf}} u - \nu_{hnf} \frac{u}{k}, \quad (2)$$

The heat equation in presence of thermal radiation, heat source and viscous dissipation is expressed in Eq. (3).

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \lambda_1 \left[u^2 \frac{\partial^2 T}{\partial x^2} + v^2 \frac{\partial^2 T}{\partial y^2} + 2uv \frac{\partial^2 T}{\partial x \partial y} \right] + v \frac{\partial u}{\partial y} \frac{\partial T}{\partial x} + u \frac{\partial v}{\partial x} \frac{\partial T}{\partial y} + v \frac{\partial v}{\partial y} \frac{\partial T}{\partial y} = \left[\frac{16\sigma^* T_\infty^3}{3(\rho C_p)_{hnf} k^*} \left(\frac{\partial^2 T}{\partial y^2} \right) + \frac{K_{hnf}}{(\rho C_p)_{hnf}} \frac{\partial^2 T}{\partial y^2} + \right. \\ \left. + \frac{Q_0 (T - T_\infty)}{(\rho C_p)_{hnf}} + \frac{\mu_{hnf}}{(\rho C_p)_{hnf}} \left(\frac{\partial u}{\partial y} \right)^2 \right], \quad (3)$$

The concentration equation has been presented in Eq. (4).

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + \lambda_2 \left[u \frac{\partial u}{\partial x} \frac{\partial C}{\partial x} + v \frac{\partial v}{\partial y} \frac{\partial C}{\partial y} + u \frac{\partial v}{\partial x} \frac{\partial C}{\partial y} + v \frac{\partial u}{\partial y} \frac{\partial C}{\partial x} \right] + 2uv \frac{\partial^2 C}{\partial x \partial y} + u^2 \frac{\partial^2 C}{\partial x^2} + v^2 \frac{\partial^2 C}{\partial y^2} = D_B \frac{\partial^2 C}{\partial y^2}, \quad (4)$$

Expressing the microorganism's density in Eq. (5).

$$u \frac{\partial n}{\partial x} + v \frac{\partial n}{\partial y} + \frac{b_m w_m}{(c_w - c_\infty)} \frac{\partial}{\partial y} \left(n \frac{\partial c}{\partial y} \right) = D_m \frac{\partial^2 n}{\partial y^2}. \quad (5)$$

The boundary conditions are specified in Eq. (6):

$$\left. \begin{aligned} u = u_w = c_0 x, \quad v = -v_w, \quad -k_{hnf} \frac{\partial T}{\partial y} = h_f (T_f - T), \quad C = C_w, \quad n = n_w \quad \text{at } y = 0, \\ u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad n \rightarrow n_\infty, \quad y \rightarrow \infty. \end{aligned} \right\} \quad (6)$$

where ρ_{hnf} denotes the density of hybrid nanofluid, B_0 represents magnetic field intensity, σ_{hnf} is electrical conductivity, C_∞ ambient concentration, σ^* be Stefan Boltzmann constant, C_w surface concentration, K_{hnf} be thermal conductivity, $(\rho C_p)_{hnf}$ specific the HNF heat capacity, h_f heat transfer coefficient, Q_0 heat source, D_B is the Brownian motion diffusivity, T_∞ ambient temperature, w_m swimming cells speed, T_w surface temperature, D_m microorganisms diffusion, b_m chemotaxis constant.

Equation (7) presents the dimensionless variables¹⁹:

$$v = -\sqrt{\nu_f c_0} f, \quad \zeta = \sqrt{\frac{c_0}{\nu_f}} y, \quad u = c_0 x f', \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad \chi = \frac{n - n_\infty}{n_w - n_\infty}. \quad (7)$$

The dimensionless system of equation has been expressed with help of Eqs. (8–11):

$$\zeta_1 (f'''' - P_0 f') - \zeta_3 S f' + \zeta_2 (f f'' - f'^2) = 0, \quad (8)$$

$$(\zeta_4 + R) \theta'' + \text{Pr} [\lambda^* \theta + \zeta_1 E c f'^2 + \zeta_5 \{f \theta' - \xi (f^2 \theta'' + f f' \theta')\}] = 0, \quad (9)$$

$$\phi'' - Sc (f \phi' + \delta_C (f f' \phi' + f^2 \phi'')) = 0, \quad (10)$$

$$\chi''' + L b f \chi'' - Pe (\phi''' (\chi + \sigma_m) + \chi'' \phi'') = 0. \quad (11)$$

The boundary conditions in dimensionless form are presented in Eq. (22):

$$\left. \begin{aligned} f(0) = \Gamma, \quad f'(0) = 1, \quad \zeta_5 \theta'(0) = -Bi (1 - \theta(0)), \quad \phi(0) = 1, \quad \chi(0) = 1, \\ f'(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 0, \quad \chi(\infty) \rightarrow 0. \end{aligned} \right\} \quad (12)$$

where $S = \sigma_f B_0^2 / \rho_f c_0$ (magnetic parameter), $P_0 = \nu_f / k c_0$ (porosity number), $\delta_E = \lambda_1 c_0$ (thermal relaxation number), $\text{Pr} = \nu_f (\rho C_p)_f / k_f$ (Prandtl number), $R = 16 \sigma^* T_\infty^3 / 3 k^* k_f$ (radiation number),

$\Gamma = v_w / \sqrt{\nu_f c_0}$ (suction parameter), $Bi = h_f / k_f \sqrt{\nu_f / c_0}$ (Biot number), $Sc = \nu_f / D_B$ (Schmidt number), $\delta_C = \varepsilon_2 c$ (thermal concentration number), $Lb = \nu_f / D_m$ (bio-convective Lewis number), $Pe = b_m w_m / D_m$ (Peclet number), $\sigma_m = n_\infty / n_w - n_\infty$ (motile difference constant), $\lambda_Q = Q_0 / c (\rho C_p)_f$ (External heat source),

$Ec = U_w^2 / (T_w - T_\infty) (C_p)_f$ (Eckert number).

Equation (13) accounts the thermal quantities:

$$\zeta_1 = \mu_{hnf} / \mu_f, \quad \zeta_2 = \rho_{hnf} / \rho_f, \quad \zeta_3 = \sigma_{hnf} / \sigma_f, \quad \zeta_5 = (\rho C_p)_{hnf} / (\rho C_p)_f, \quad \zeta_4 = K_{hnf} / K_f. \quad (13)$$

The dimensionless form of skin friction coefficient, Nusselt number, Sherwood number and motile density number are expressed in Eq. (14) as¹⁹:

$$\left. \begin{aligned} \text{Re}_x^{1/2} C_f &= \zeta_1 (f''(0)), \\ \text{Re}_x^{-1/2} Nu &= -(\zeta_4 + R) \theta'(0), \\ Sh(\text{Re}_x)^{-1/2} &= -\phi'(0), \\ Nn(\text{Re}_x)^{-1/2} &= -\chi'(0). \end{aligned} \right\} \quad (14)$$

Thermal properties of hybrid nanofluid

In Table 1, the numerical values are expressed for MoS_2 , ZnO and $\text{SAE10W} - 30$ engine oil. Table 2 focused to development of mathematical relations for expressing thermal quantities for mono and hybrid nanofluids. Table 3 proposed understanding of different shape features. It should be observed that the impact of nanoparticle

Physical Properties	MoS_2	ZnO	$\text{SAE10W} - 30$
$\rho (\text{kgm}^{-3})$	5060	5600	1260
$C_p (\text{Jkg.k})$	397.21	495.2	2415
σ	0.029	0.0005	0.95
$\kappa (W.(m.k))$	904.4	13	0.286
Pr	-	-	50

Table 1. Numerical variation of MoS_2 , ZnO and $\text{SAE10W} - 30$ engine oil¹⁹.

Properties	Nanofluid
Density(ρ), Kg/m^3	$\rho_{nf} = (1 - \phi_1) \rho_{bf} + \phi_1 \rho_{s1}$
Heat Capacity(C_p), J/K	$(\rho C_p)_{nf} = (1 - \phi_1) (\rho C_p)_{bf} + \phi_1 (\rho C_p)_{s1}$
Viscosity(μ), Ns/m^2	$\mu_{nf} = \frac{\mu_{bf}}{(1 - \phi_1)^{2.5}}$
Thermal Conductivity(k_f), $W/m K$	$\frac{k_{nf}}{k_{bf}} = \frac{k_{s1} + (s - 1) k_{bf} - (s - 1) \phi_1 (k_{bf} - k_{s1})}{k_{s1} + (s - 1) k_{bf} + \phi_1 (k_{bf} - k_{s1})}$,
Properties	Hybrid nanofluid
Density(ρ), Kg/m^3	$\rho_{hnf} = ((1 - \phi_1) \rho_f + \phi_1 \rho_{s1}) (1 - \phi_2) + \phi_2 \rho_{s2}$
Heat Capacity(C_p), J/K	$(\rho C_p)_{hnf} = (1 - \phi_1) (\rho C_p)_{bf} + \phi_2 (\rho C_p)_{s2} + (1 - \phi_2) \phi_1 (\rho C_p)_{s1}$
Viscosity(μ), Ns/m^2	$\mu_{hnf} = \mu_{bf} (1 - \phi_1)^{-5/2} (1 - \phi_2)^{-5/2}$
Thermal Conductivity(k_f), $W/m K$	$\frac{k_{hnf}}{k_{nf}} = \frac{k_{s2} + (s - 1) k_{nf} - (s - 1) \phi_2 (k_{nf} - k_{s2})}{k_{s2} + (s - 1) k_{bf} + \phi_2 (k_{nf} - k_{s2})}$

Table 2. Thermo-physical relations for nanofluid and hybrid nanofluid¹⁹.

Particle form	Shape factor
Sphere	3.0
Platelet	5.7
Blade	8.6
Cylinder	4.9
Brick	3.7

Table 3. Shape structure of different nanoparticles^{22,23}.

configuration in current analysis is analyzed through effective thermo-physical correlations, especially the effective thermal conductivity model. This formulation does not explicitly account the particle orientation, anisotropic, particle aggregation, Brownian diffusion or surface roughness features. Therefore, the analyzed impact of nanoparticle-shape should be interpreted within scope of proposed effective medium approach.

Analysis of shape features

Table 3 incorporates the different nanoparticles configurations along with associated shape factor, which are used to analyse the features of particle shape on thermo-physical features of HNF. The shape factor is the dimensionless number that features the fluctuation of particle from an ideal spherical configuration. For spherical nanoparticle, the value of shape factor is 3.0, claiming the symmetric and baseline configuration. When structure become complex and elongated, the shape factor enhances, as examined for brick (3.7), cylinder (4.9), platelet (5.7), and blade (8.6) shapes. Leading shape factor confirm enhanced surface area to volume ratio with strengthened anisotropic consequences, which increases the energy transport dynamics within fluid. The non-spherical particles like blades and platelets signifies more thermal impact as compared to spherical particles. Such diversion in shape attributes an important character in controlling of heat transfer and fluid behaviour within HNF systems.

Numerical computations

For simulating the solution of modelled equations, we have implemented the most convergent numeric technique namely shooting method. The motivated shooting technique is preferred due to computational accuracy, peak efficiency and simplicity. Shooting method is famous numerical scheme, effectively solve the boundary value problem. In this method, initial conditions are estimated where the resulting system is integrated with help of numerical solver like Runge–Kutta technique. The compiled solution is compared with the prescribed conditions near the surface. If boundary conditions are not fulfilled, the initial guesses are refined via Newton-Raphson technique.

The conversion of higher order system into first order approximation is based on choice of appropriate variables which are defined in Eq. (15):

S	Hayat et al. ³⁴	Present results
0.5	− 1.224747	− 1.224744
1	− 1.414217	− 1.414215
1.5	− 1.581147	− 1.581146
2.0	− 1.732057	− 1.7320575

Table 4. Results for $f''(0)$ when $\zeta_1 = \zeta_2 = \zeta_3 = 1$ and $P_0 = 0$.

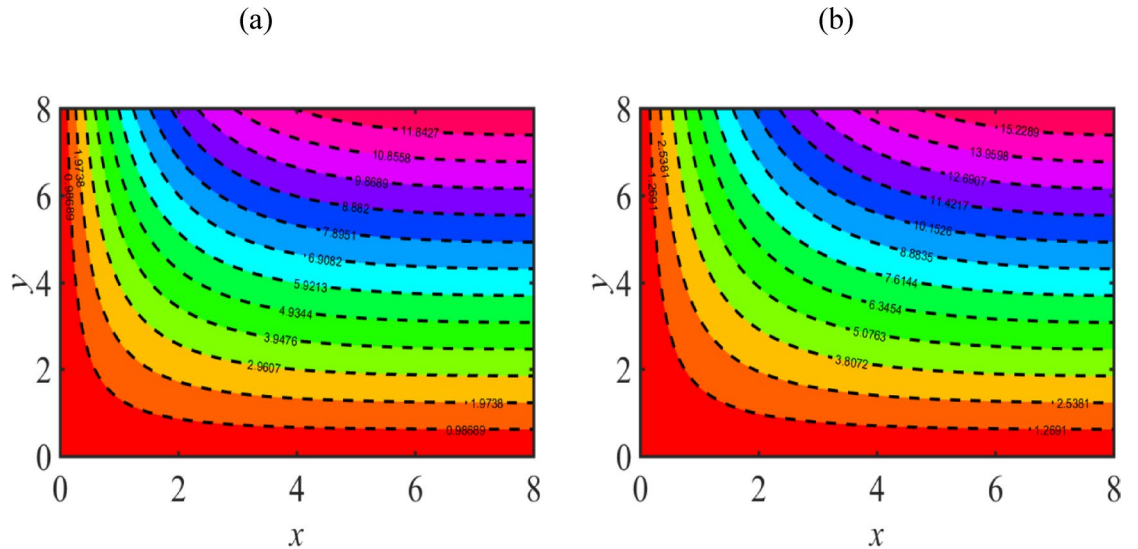


Fig. 2. (a-b): Streamlines for velocity profile when (a) $S = 0.0$ (b) $S = 0.2$.

$$\left. \begin{aligned} f &= a_1, f' = a_2, f'' = a_3, f''' = a_3', \\ \theta &= a_4, \theta' = a_5, \theta'' = a_5', \\ \phi &= a_6, \phi' = a_7, \phi'' = a_7', \\ \chi &= a_8, \chi' = a_9, \chi'' = a_9'. \end{aligned} \right\} \quad (15)$$

The initial values problem is expressed in Eqs. (16–19):

$$a_3' = \frac{\zeta_1 P_0 a_2 + \zeta_3 S a_2 - \zeta_2 (a_1 a_3 - a_2^2)}{\zeta_1}, \quad (16)$$

$$a_5' = \frac{-\text{Pr} [\lambda^* a_4 + \zeta_1 E c a_2^2 + \zeta_5 \{a_1 a_5 - \xi a_1 a_2 a_5\}]}{(\zeta_4 + R) - \text{Pr} \xi \zeta_5 a_1^2} \quad (17)$$

$$a_7' = \frac{S c [-a_1 a_7 + \delta_C a_1 a_2 a_7]}{(1 - S c \delta_C a_1^2)} \quad (18)$$

$$a_9' = P e [a_7' (a_8 + \sigma) + a_9 a_7] - L b a_1 a_9. \quad (19)$$

Equation (12) entertained the transformed boundary conditions :

$$\left. \begin{aligned} a_1(0) &= \Gamma, a_2(0) = 1, \zeta_5 a_5(0) = -B i [1 - a_4(0)], a_6(0) = 1, a_8(0) = 1, \\ a_2(\infty) &\rightarrow 0, a_4(\infty) \rightarrow 0, a_6(\infty) \rightarrow 0, a_8(\infty) \rightarrow 0. \end{aligned} \right\} \quad (20)$$

The accuracy of solution is justified up to 10^{-4} . The validation of simulated numerical data is confirmed in Table 4. The results are compared with contribution of Hayat³⁴ under limiting case where a convincing agreement is observed between both studies.

Results and discussion

In order to explore the physical explanation of problem, comparative results are examined for different flow parameters for mono nanofluid (MNF) ($\text{MoS}_2/\text{SAE10W-30}$) and hybrid nanofluid (HNF) ($\text{MoS}_2\text{-ZnO}/\text{SAE10W-30}$). The streamlines are plotted in Fig. 2(a-b) for analysing the flow pattern for velocity profile. The flow behaviour is observed for both hydromagnetic and magnetohydrodynamics cases. For both cases, a smooth

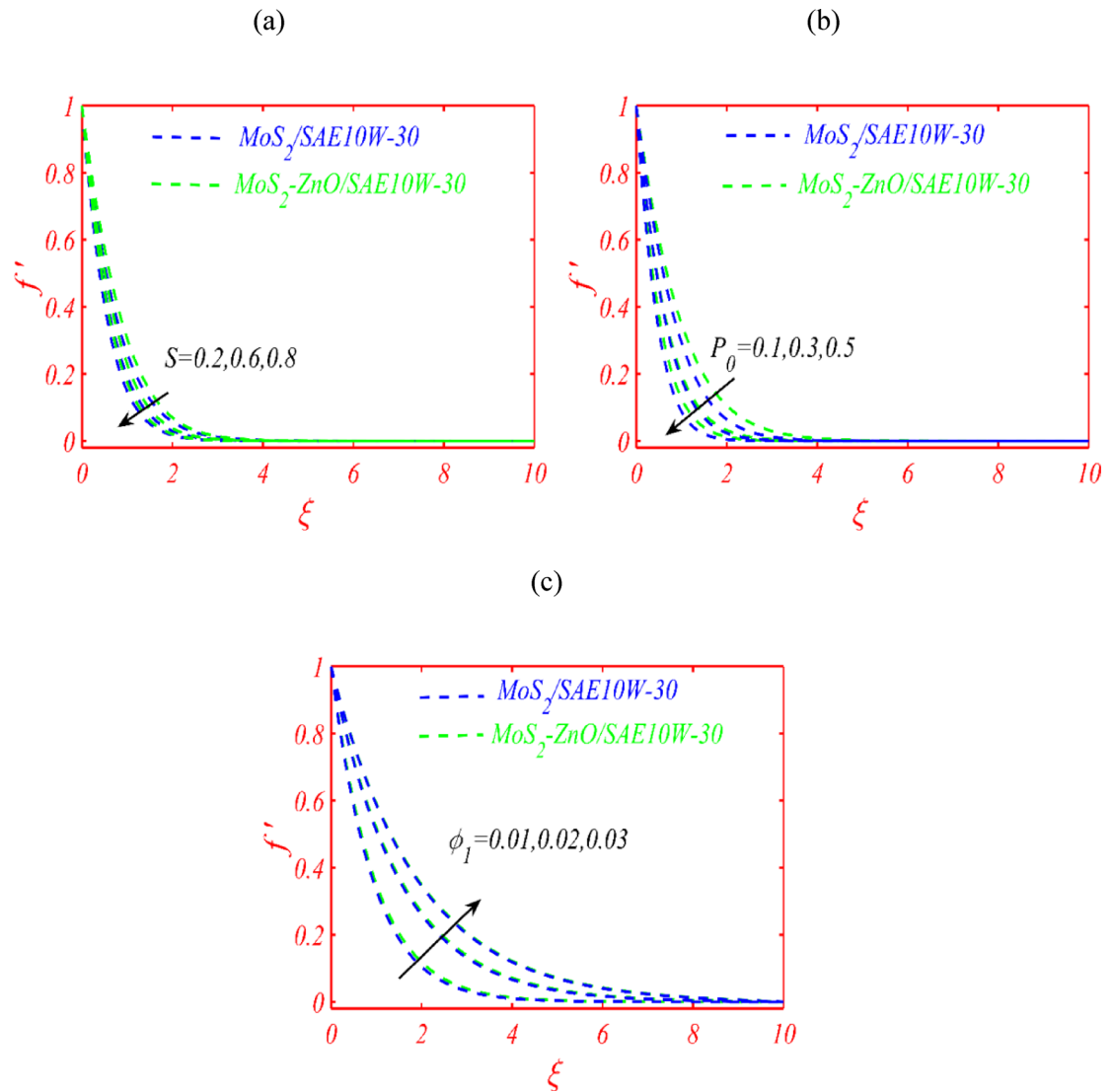


Fig. 3. (a–c): Analysis of f' for (a) S when $P_0 = 0.5, \phi_1 = \phi_2 = 0.02$ (b) P_0 when $S = 0.3, \phi_1 = \phi_2 = 0.02$ (c) ϕ_1 when $S = 0.3, P_0 = 0.5$.

and laminar velocity profile has been observed. Figure 3(a–c) has been prepared to check the effects of magnetic parameter S , porosity parameter P_0 and nanoparticles volume fraction ϕ_1 on velocity profile f' . Figure 3(a) concentrates to visualization of f' due to increasing impact of magnetic parameter S . A decreasing change is observed in assessment of f' due to larger S . In fact, larger S causing the Lorentz force providing resistance to the fluid motion. The reduction behaviour is relatively slower for hybrid nanofluid suspension ($\text{MoS}_2\text{-ZnO/SAE10W-30}$). Such trend is good agreement with the analysis of Mebarek-Oudina et al.², Hayat et al.⁴, and Qamar et al.¹⁸, where it has been claimed that the Lorentz force induces by an applied magnetic force suppresses the fluid movement and thickens the momentum boundary layer. Current investigation further accomplishes that the velocity decrement is comparatively weaker for $\text{MoS}_2\text{-ZnO}$ hybrid nanomaterial than for mono nanofluid, presenting enhanced momentum transport due to synergistic interaction of dual nanoparticles. Similar findings are observed when effects of porosity parameter P_0 have been endorsed (Fig. 3(b)). The reduction in profile of f' is subject to presence of porous medium, providing resistance to the flow. The presence of porous matrix induces additional drag force which suppress the momentum transport. From comparative point of view, the MNF pronounced more effective velocity reduction as compared to HNF. Figure 3(c) elaborates the significance of nanoparticles volume fraction ϕ_1 on f' where increasing observations have been noticed.

Figure 4(a) determines the prediction of temperature profile θ due to radiated parameter R . Larger R leads to enhances the radiated heat transport due to electromagnetic waves. In current investigation, the role of base fluid is specified with SAE10W-30 engine oil for which radiated effects plays important role in maintaining the efficiency in automobile systems. In practical applications, such behaviour is relevant to solar thermal systems, peak-temperature lubrication of engine components, metallurgical processes, cooling of nuclear reactor and radiative heat shielding technologies. The improvement in thermal profile with enhancing radiation parameter is consistent with the numerical contribution of Wang et al.¹, Tlili⁹, and Alam et al.¹⁰, where the radiation

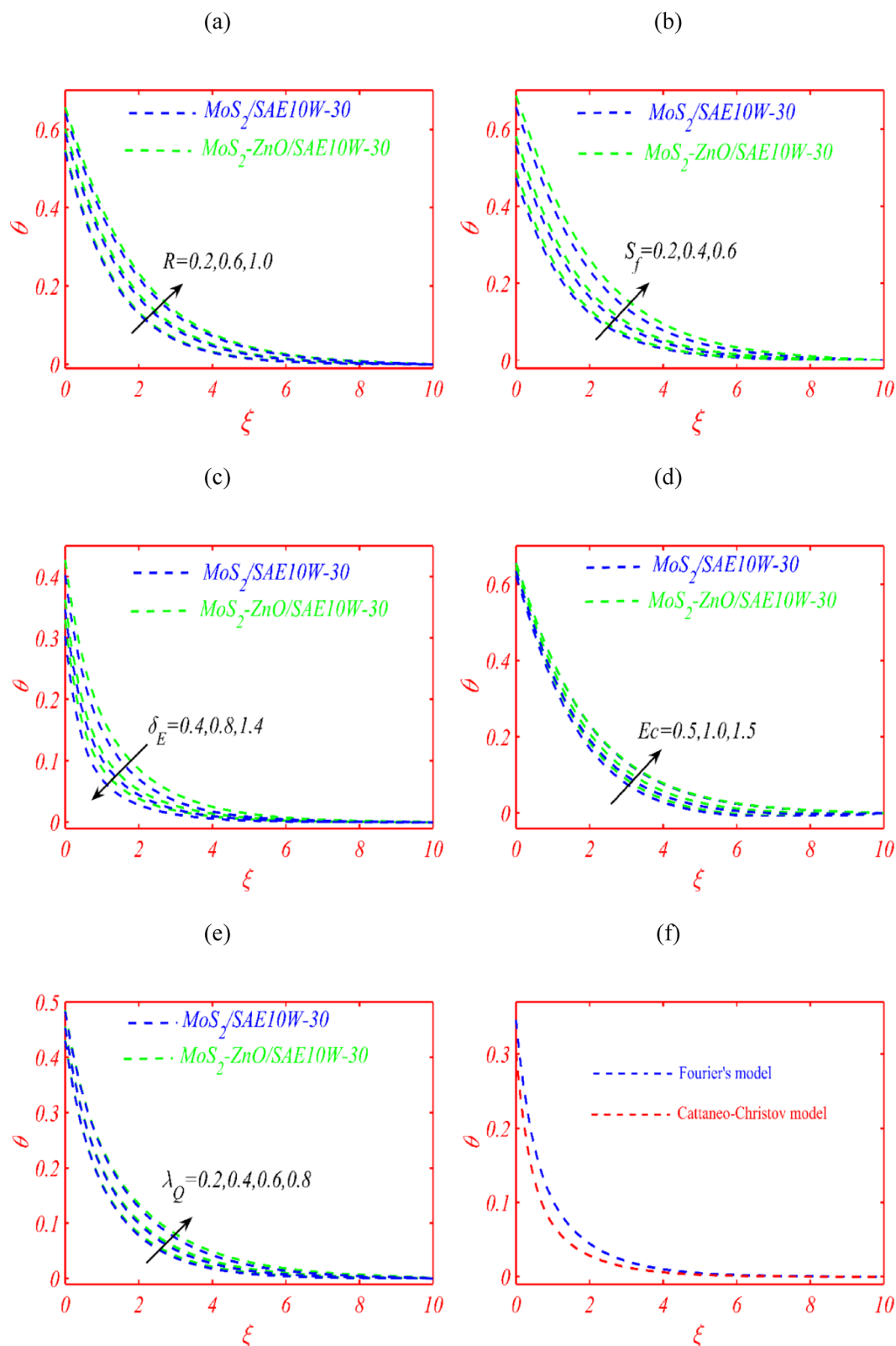


Fig. 4. (a–f): Analysis of θ for (a) R when $P_0 = 0.5, S_f = 0.2, \delta_E = 0.6, \lambda_Q = 0.4, S = 0.4, Ec = 0.6, \phi_1 = \phi_2 = 0.02$ (b) S_f when $P_0 = 0.5, S_f = 0.2, \delta_E = 0.6, Ec = 0.6, \lambda_Q = 0.4, S = 0.4, \phi_1 = \phi_2 = 0.02$ (c) δ_E when $P_0 = 0.5, S_f = 0.2, Ec = 0.6, \lambda_Q = 0.4, S = 0.4, \phi_1 = \phi_2 = 0.02$ (d) Ec when $P_0 = 0.5, S_f = 0.2, \delta_E = 0.6, \lambda_Q = 0.4, S = 0.4, \phi_1 = \phi_2 = 0.02$ and (e) λ_Q when $P_0 = 0.5, Ec = 0.6, S_f = 0.2, \delta_E = 0.6, S = 0.4, \phi_1 = \phi_2 = 0.02$ (f) comparative analysis for classical Fourier heat conduction model ($\delta_E = 0.0$) and Cattaneo-Christov model ($\delta_E = 0.4$) when $P_0 = 0.5, \lambda_Q = 0.4, S_f = 0.2, \delta_E = 0.6, \lambda_Q = 0.4, S = 0.4, \phi_1 = \phi_2 = 0.02$.

effects was presented to intensify the energy transport within the boundary layer. Current analysis extends such investigations by assuming the engine oil-based HNF together with updated Fourier's model. Figure 4(b) incorporates the evaluation of θ due to shape factor S_f . The heat transfer shows growing behaviour due to higher S_f . In fact, S_f is associated to the nanoparticles geometry impacting the heat transport dynamics within fluid system. The non-spherical particles like cylinders or blades incorporates peak shape factor, which enriches the effective surface area for thermal interaction within base liquid. The impact of nanoparticle morphology agrees well with analysis of Hussain et al.²², Sreelakshmi et al.²⁴, Sandeep et al.²⁵, Hajlaoui et al.²⁶, and Ghachem et al.²⁸, where it was concluded that non-spherical nanoparticles present superior heat transfer due to larger effective surface area and impressive thermal interactions. Current findings further extend these investigations by demonstrating that the combined incorporation of MoS_2 -ZnO/SAE10W-30 hybrid nanofluid, together with shape-dependent thermal conductivity, presents a more pronounced increment in thermal transport than mono nanofluids. Furthermore, the implementation of the Cattaneo-Christov heat flux model incorporates the impact of finite thermal relaxation on shape-dependent heat transfer, providing a more realistic prediction of heat transport for high-temperature engineering and lubrication systems. Figure 4(c) aims to justify the profile of θ against higher thermal relaxation number δ_E . A decrement in profile of θ is resulted due to δ_E . The reduction in heat transfer due to δ_E is physically associated to finite time required for heat flux to adjust the temperature gradient in fluid flow. In traditional Fourier's model, an instantaneous response of heat flux is observed while utilization of Cattaneo-Christov model accounts the delay in heat propagation. The slower temperature profile due to thermal relaxation number is consistent with the Cattaneo-Christov transport claimed by Said et al.¹⁹ and Ghachem et al.²⁸, confirming that finite thermal waves propagation delays reduces the thickness of thermal boundary layer. Figure 4(d) shows an increment in θ due to Eckert number Ec . Such outcomes are physically conversion of kinetic energy into thermal energy due to applications of dissipative force. The Eckert number represents the ratio of kinetic energy to enthalpy difference, and distinguish the effects of frictional heating in boundary layer. The results compiled in Fig. 4(e) provides the role of heat source parameter λ_Q on θ where increasing effects are noted. The presence of heat source successfully provides extra thermal energy to the system

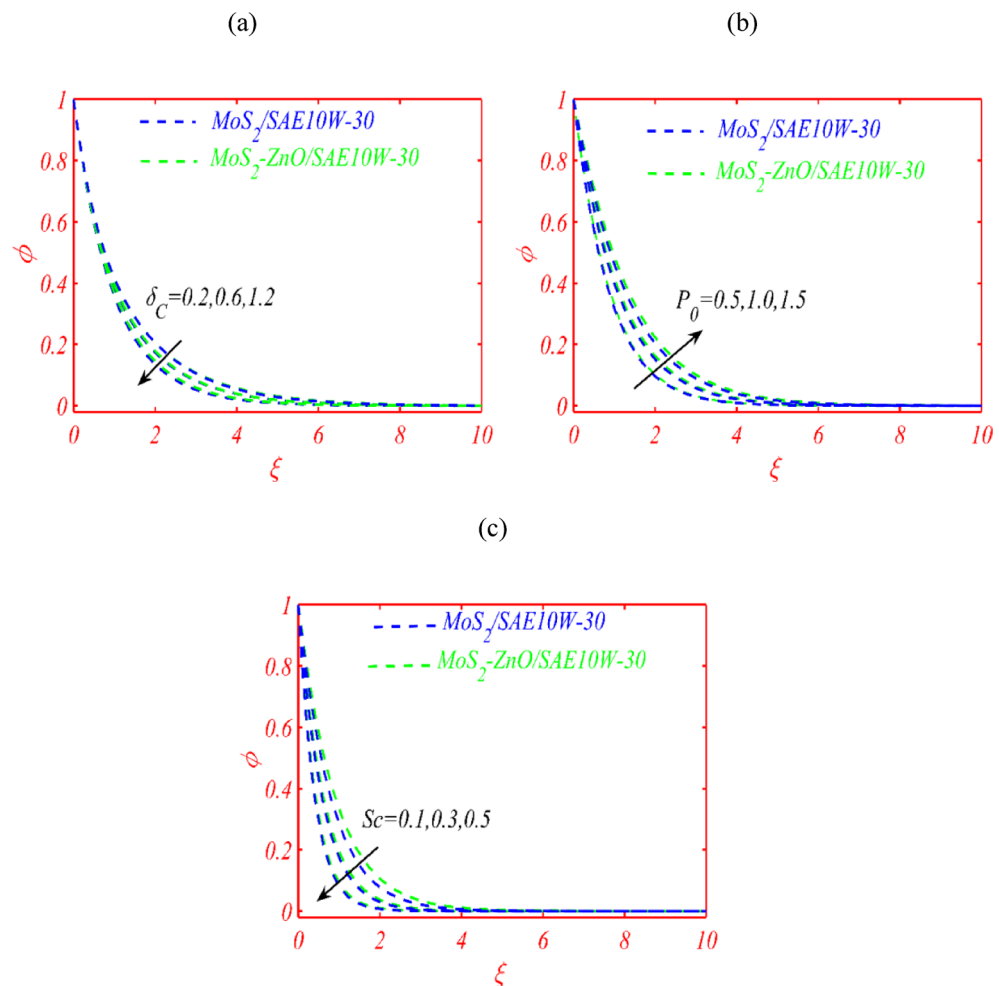


Fig. 5. (a–c): Analysis of ϕ for (a) δ_C when $P_0 = 0.5, Sc = 0.4, S_f = 0.2, \delta_E = 0.6, \lambda_Q = 0.4, S = 0.4, \phi_1 = \phi_2 = 0.02$ (b) P_0 when $Sc = 0.5, \delta_C = 0.4, S_f = 0.2, \delta_E = 0.6, \lambda_Q = 0.4, S = 0.4, \phi_1 = \phi_2 = 0.02$ (c) Sc when $P_0 = 0.5, S_f = 0.2, \delta_E = 0.6, \lambda_Q = 0.4, \delta_C = 0.4, S = 0.4, \phi_1 = \phi_2 = 0.02$.

which helps in increasing the thermal transport phenomenon. Figure 4(f) demonstrates the comparative analysis of classical Fourier heat conduction model and Cattaneo-Christov model. The control of heat transfer is more impressive with implementation of Cattaneo-Christov model.

The discussion of concentration profile ϕ in view of important flow parameters like concentration number δ_C , porosity parameter P_0 and Schmidt number Sc has been disclosed in Fig. 5(a-c). Figure 5(a) explaining the impact of concentration number δ_C on ϕ with comparative impact for both MNF and HNF. The concentration for both HNF and MNF declines gradually for prominent impact of δ_C . The slower change in ϕ due to δ_C is associated to finite time necessitated for mass diffusion to respond the concentration gradients in fluid flow. The mass diffusion delays and controlling the driving force for nanoparticles transport from higher to lower concentration gradient regimes. The diffusion process becomes less effects and overall concentration field reduces. Figure 5(b) explores the prediction of ϕ due to porosity number P_0 where increasing concentration profile has been noticed. Such outcomes are physically linked to enhancement of permeability of medium which encourages the mass transport process. Figure 5(c) elucidates the investigation of Schmidt number Sc on ϕ where decreasing outcomes have been observed. The reduction impact in assessment of ϕ due to Sc corresponds to low mass diffusivity.

Figure 6(a-c) characterizes the results for microorganisms profile χ by increasing bioconvective Lewis number Lb , Peclet number Pe and microorganisms difference parameter σ . The enhancement in Lb leads to decrement of χ . Larger Lb is associated to relative dominance of momentum and thermal transport over the microorganisms diffusivity. The microorganism diffusivity become weaker against larger Lb which leads to reduction of χ . The reduction in χ due to variation of Peclet number Pe is observed due to low motile diffusivity. Similar findings for assessment of χ have been simulated in case of microorganisms difference parameter σ . These findings are consistent with the bioconvection investigations of Shah et al.²⁹, Hussain et al.³⁰, Alshammari et al.³¹, and Ahlawat et al.³², where it was claimed that the Peclet number and bioconvective Lewis plays important role in suppressing the microorganism distribution due to reduced microorganism diffusivity.

Figure 7(a-c) illustrates a 3D visualization of velocity f' , temperature θ and concentration profiles ϕ .

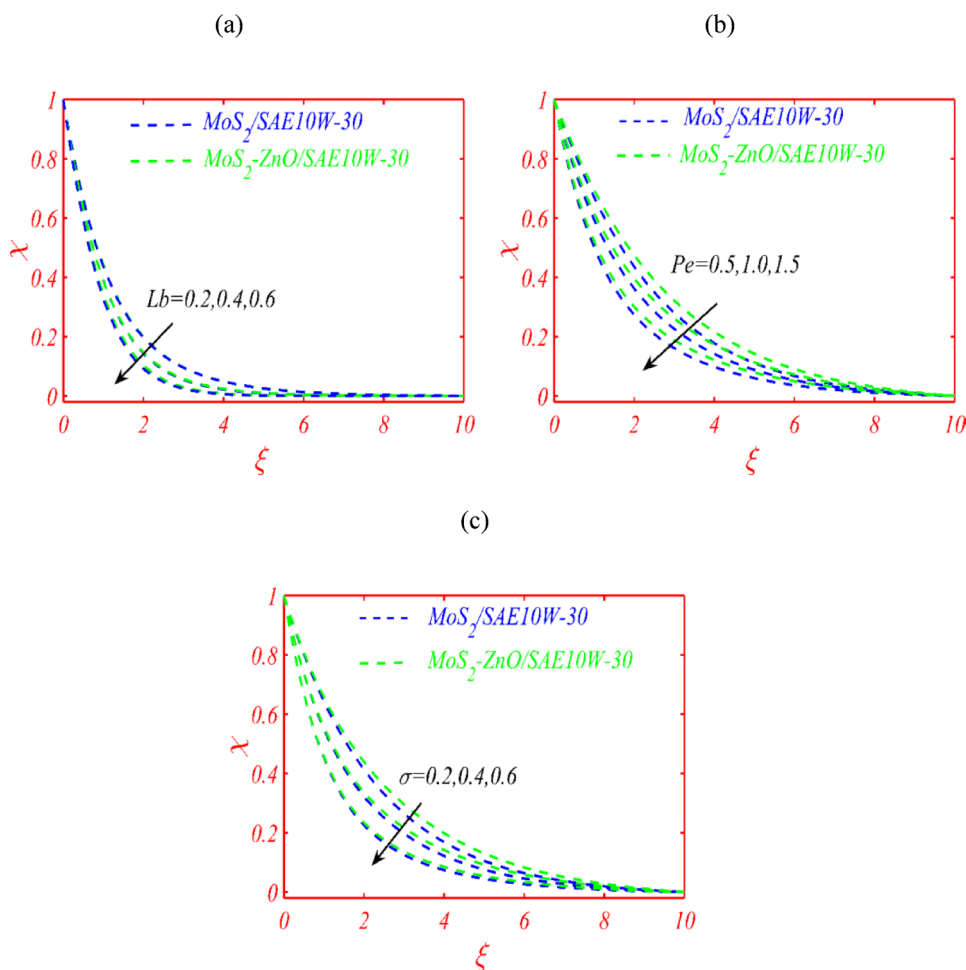


Fig. 6. (a–c): Analysis of χ for (a) Lb when $Pe = 0.6, P_0 = 0.5, S_f = 0.2, \delta_E = 0.6, \lambda_Q = 0.4, \delta_C = 0.4, S = 0.4, \phi_1 = \phi_2 = 0.02$ (b) Pe when $P_0 = 0.5, Lb = 0.7, S_f = 0.2, \delta_E = 0.6, \lambda_Q = 0.4, \delta_C = 0.4, S = 0.4, \phi_1 = \phi_2 = 0.02$ (c) σ when $Lb = 0.7, P_0 = 0.5, S_f = 0.2, \delta_E = 0.6, \lambda_Q = 0.4, \delta_C = 0.4, S = 0.4, \phi_1 = \phi_2 = 0.02$.

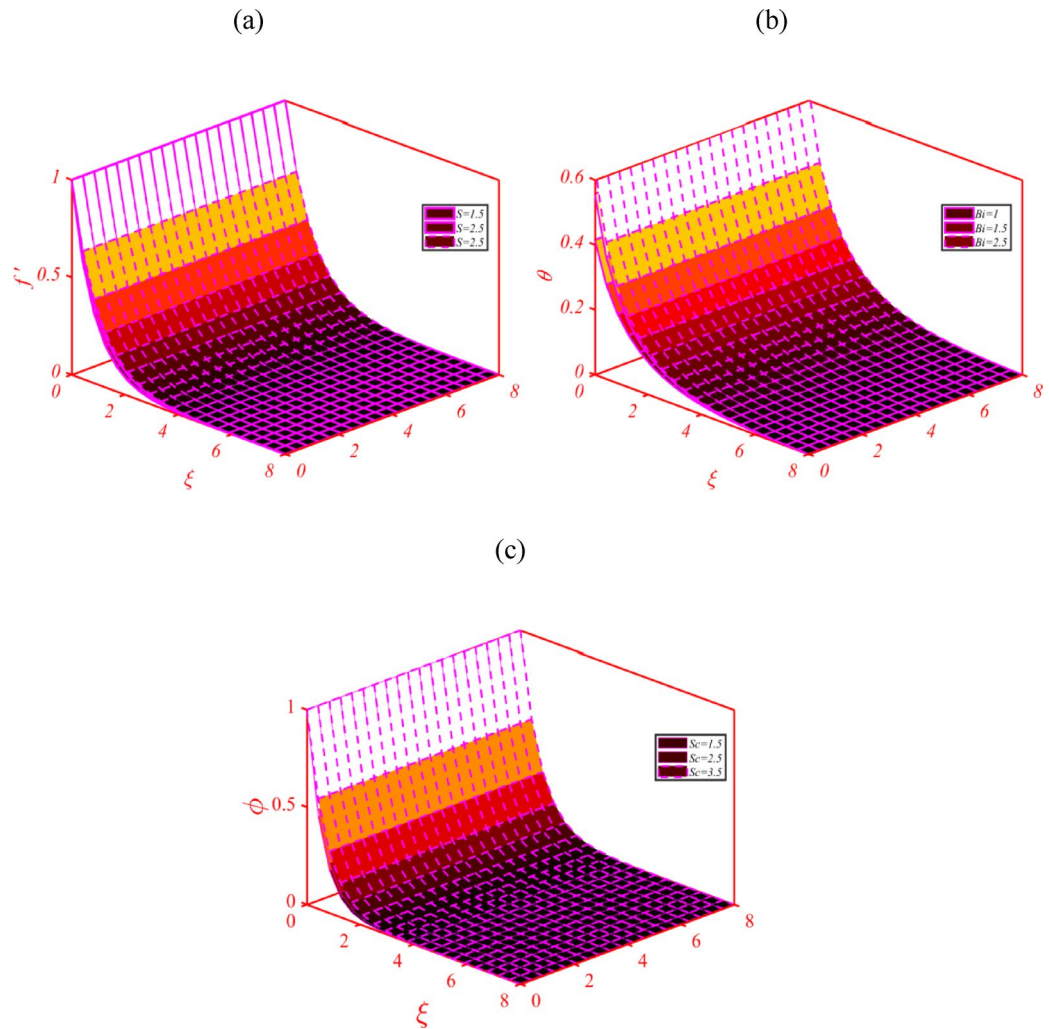


Fig. 7. (a–c): A 3D analysis for (a) f' (b) θ and (c) ϕ .

Figure 8(a) aims to analyse the trend of skin friction coefficient $Re_x^{1/2} C_f$ versus magnetic parameter S while variation in porosity parameter P_0 has been assigned. The profile of $Re_x^{1/2} C_f$ reduces by varying P_0 . The results proposed in Fig. 8(b) determines the variation of Nusselt number $Re_x^{1/2} Nu$ against Bi when role of thermal relaxation number δ_E is involved. The Nusselt number reduces due to higher δ_E . In order to evaluate the analysis for Sherwood number $Re_x^{-1/2} Sh$ against Schmidt number Sc , Fig. 8(c) is plotted. It is claimed that Sherwood number boosted against Sc by enhancing δ_C . Figure 8(d) preserving the simulations for motile density number $Re_x^{-1/2} Nn$ against Peclet number Pe while Lb gets vary. A decreasing change in profile of $Re_x^{-1/2} Nn$ has been observed due to Lb .

To further present the novelty of proposed model, a comparative assessment with relevant investigations from the available literature has been presented in Table 5. The comparison accomplishes that current study extends the previous investigations by simultaneously incorporating the hybrid nanofluid (MoS_2 -ZnO/SAE10W-30), nanoparticle shape, Cattaneo–Christov heat and mass flux models, bioconvection, and a comparative analysis with mono nanofluid under magnetized porous flow conditions.

Conclusions

Thermal transport of hybrid nanofluid (MoS_2 -ZnO/SAE10W-30) with assessment of shape features is studied. The hybrid nanofluid includes the suspension of molybdenum disulphide (MoS_2) and zinc oxide (ZnO) nanoparticles with SAE10W-30 engine oil. The numerical treatment of developed equations is based on implementation of shooting scheme. Novel observations are:

- The velocity profile decays due to porous medium while higher nanoparticles volume fraction enhances the velocity profile.
- The heat transfer increases with Eckert number, nanoparticles shape feature parameter and radiation number, promoting the heating applications in automobile industries.

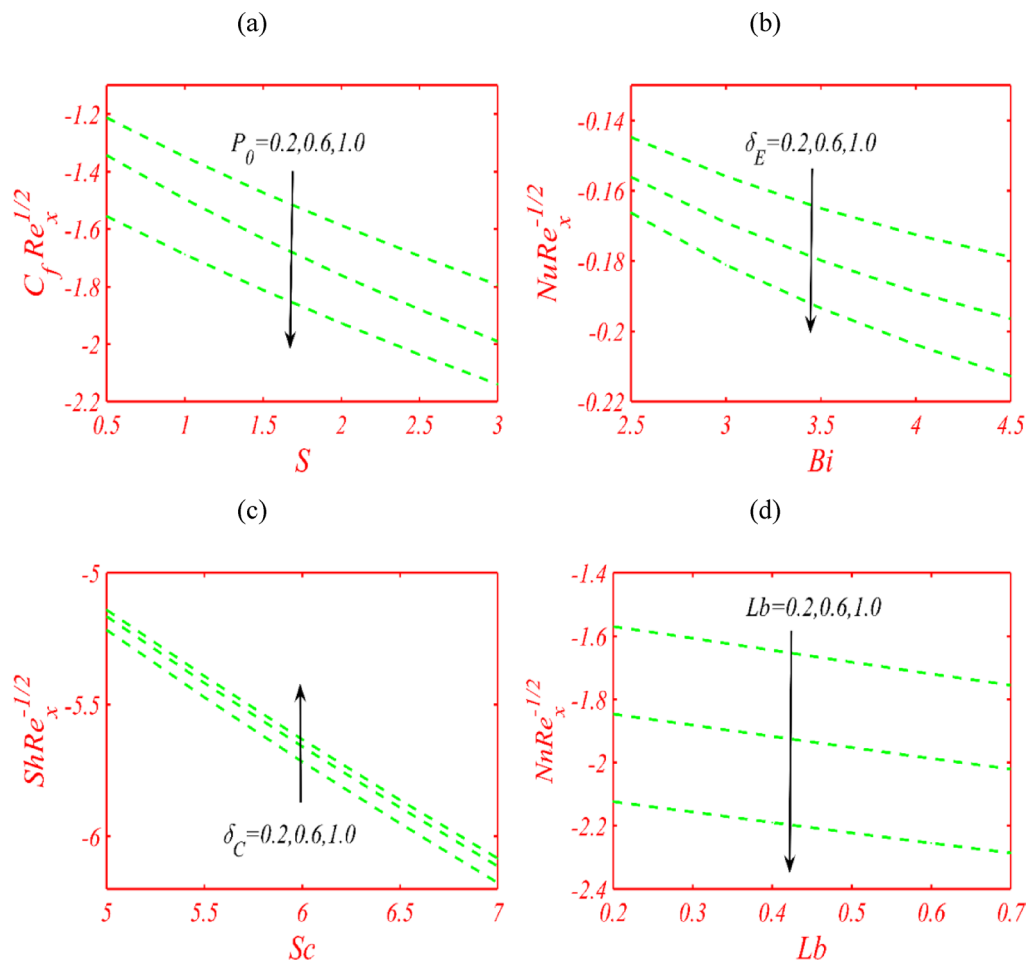


Fig. 8. (a–d): (a): Variation of $Re_x^{1/2} C_f$ versus S for P_0 (b) variation of Nusselt number $Re_x^{-1/2} Nu$ against Bi for δ_E (c) variation of Sherwood number $Re_x^{-1/2} Sh$ against Sc for δ_C and (d) variation of motile density number $Re_x^{-1/2} Nn$ against Pe for Lb .

References	Model/Fluid	Major Findings	Advancement in the Present Study
Wang et al. ¹	Radiative MoS ₂ nanofluid	Radiation enhances thermal transport.	Confirms this trend and extends it to MoS ₂ –ZnO/SAE10W-30 hybrid nanofluid with shape-factor analysis.
Mebarek-Oudina et al. ²	MHD hybrid nanofluid in porous medium	Magnetic field and porous medium suppress velocity.	Similar observations are obtained while additionally incorporating bioconvection and non-Fourier heat/mass transport.
Hayat et al. ⁴	MHD hybrid nanofluid	Darcy porous medium reduces momentum transport.	Present work further compares mono and hybrid nanofluids and quantifies improved transport characteristics of HNF.
Hussain et al. ²²	Shape-dependent nanofluid	Nanoparticle morphology significantly influences heat transfer.	Extends the analysis by combining shape effects with hybrid nanofluid, thermal relaxation, and engine-oil base fluid.
Sreelakshmi et al. ²⁴	Mono and hybrid nanofluids with shape effects	Hybrid nanofluid provides superior thermal enhancement.	Present results confirm this behavior and further investigate Cattaneo–Christov heat and mass flux with bioconvection.
Hajlaoui et al. ²⁶	Magnetized hybrid nanofluid	Thermal radiation and viscous dissipation increase temperature.	Similar behavior is observed while incorporating concentration transport, microorganism dynamics, and comparative MNF–HNF analysis.
Alshammari et al. ³¹	Bioconvection hybrid nanofluid	Radiation enhances temperature and bioconvection affects microorganism distribution.	Extends the study through dual relaxation (heat and mass), nanoparticle shape factor, and engine-oil-based hybrid nanofluid.
Present Study	MoS ₂ –ZnO/SAE10W-30 Hybrid Nanofluid	Velocity decreases with magnetic field and porosity; temperature increases with radiation, Eckert number, and nanoparticle shape factor; concentration decreases with Schmidt number and concentration relaxation; microorganism density decreases with bioconvective Lewis and Peclet numbers. HNF consistently exhibits better thermal performance than MNF.	Novel integration of MoS ₂ –ZnO/SAE10W-30 hybrid nanofluid, nanoparticle shape effects, Cattaneo–Christov heat and mass transport, bioconvection, and direct comparison with mono nanofluid in a single numerical framework.

Table 5. Comparative assessment between the present study and existing literature.

- Upon increasing the shape factor from sphere to blade indicates an enhanced thermal capability due to increased anisotropy, larger interfacial interaction and surface roughness between the base fluid and nanoparticles.
- The heat transfer declines for larger thermal relaxation parameter, demonstrating the significance of modified Fourier's heat flux delay in thermal system.
- The concentration profile attributes a dual behaviour, declining with Schmidt number and concentration relaxation parameter due to low mass diffusivity.
- The microorganism profile is found to declines by increasing the bioconvective Lewis number, reflecting reduces microorganism diffusivity and weaker bioconvective transport features.
- The comparative assessment between the HNF and MNF indicates the HNF ($\text{MoS}_2\text{-ZnO/SAE10W-30}$) exhibit stronger thermal impact and improved transport features as compared to mono nanofluid ($\text{MoS}_2/\text{SAE10W-30}$). Such enhancement is attributed to synergistic interaction of dual nanoparticles, which intensifies the energy management and modifies the mass diffusion behavior more effectively.

Future research directions

This model can be trained by using the machine learning simulations to improve the accuracy of model. The accuracy of model can be further updated by incorporating the sensitivity analysis to strengthen the robustness of the numerical predictions. Future investigations may further account the multiscale models to inspect these dynamics in comprehensive detail.

Limitations of the present study

This study is based on consideration of ideal flow assumptions by ignoring the effects of external forces. Therefore, there may exists minor deviation between the current theoretical data and experimental-based simulation.

Data availability

All data generated or analysed during this study are included in this published article.

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Author contributions

All authors contribute equally.

Declarations

Competing interests

The authors declare no competing interests.

Additional information

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