Strong Edge Coloring in Tree-Like Networks

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***Abstract*—A strong edge coloring of a graph is a proper edge coloring such that no two edges within a distance of two share the same color. The strong chromatic index of the graph G is denoted by $\chi^{'}(G)$. Strong edge coloring is a variation of coloring problems and it has practical uses in various fields such as wireless communication, resource allocation, VLSI circuit design, and task scheduling. It is an extension of the traditional edge coloring that is used in communication networks, but it is more complex as it requires that no two edges within a distance of two share the same color, an idea that is implemented in different ways to allow for much more efficient use of spectrum. In our work, we design a heuristic algorithm based on edge degrees and recursive connectivity levels that gives priority to edge coloring, as a result of which we manage to minimize the color usage that ensures conflict-free color assignments with the smallest number of colors. This work is aimed at presenting an interdisciplinary prospect that blends formal mathematical studies on combinatorics with the practical use of fractal graph colorings, the theory of partitioning and network models with structure.**

Keywords: **Chromatic index, Graph coloring, Edge coloring, , Networks, Visualization,Graph Structure, Channel Assignment .**

I. Introduction

Edge coloring is an important problem in graph theory which has applications in network theory, scheduling algorithms and in resource allocation. This constraint guarantees that each edge color could not be assigned concurrently thus having a strong implication on problems in the real world as in the assignment of wireless channels in a wireless network, optical network design, CPU task scheduling and the optimization of VLSI circuit layout. Strong edge coloring can be used for many applications that are listed below: Wireless channel assignment The distinctive color assignments allow only small amounts of interference between adjacent connections while optical wavelength routing the wavelength assignments can be given to fiber-optic channels to avoid collisions and maximize transmission efficiency. Task scheduling can be used to help minimize conflict when scheduling tasks across multiple distributed systems. Recursive Rhombus Carpet edge coloring can also be used in VLSI circuit design, for optimizing the arrangement of electrical connection terminations, reducing crosstalk and power consumption. Its application can also be made to traffic signal optimization, for optimizing non-overlapping transport schedules that can improve traffic flow and prevent congestion. Strong edge coloring can be applied in biological network analysis where it is used to model genetic and protein interaction networks, for example to ensure that related but interdependent biological pathways do not interfere with each other. The structure of the Recursive Rhombus Carpet may also serve in community detection in social networks, where distinct color assignments provide information about clusters in complex network structures, and in transportation logistics, where route optimization through different color assignments can help to avoid congestion and increase system efficiency. .

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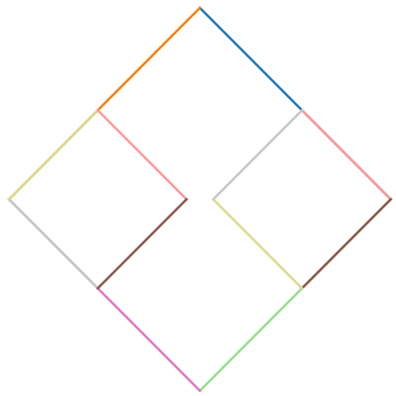
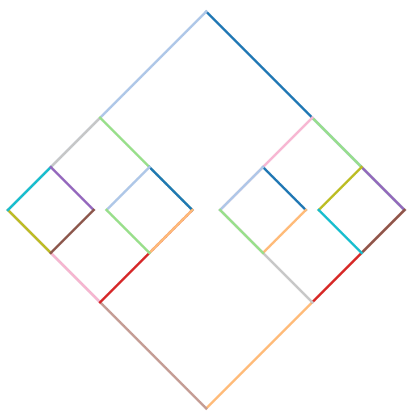
II. Literature Review

Edge colouring is a basic problem in the graph theory in which each edge of the graph is assigned with a colour such that no two adjacent edges shares the same colour. The minimum number of colours needed to exactly colour the edges of a graph G is known as its chromatic index. One of the most important results in the edge colouring is the Vizing's theorem. While the traditional edge colouring ensures that no two adjacent edges have the same colour, the strong edge colouring imposes a strict condition, where each edge at a distance less than

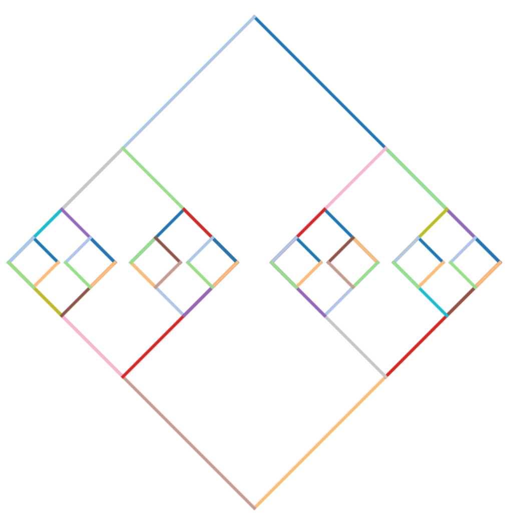
B. Experimental Results



(a) Level 2



(b) Level 3



(c) Level 4

Fig. 4: Strong edge colouring for different levels

V. Conclusions

In this paper, we obtained the sharp lower bound for strong coloring of tree-Like networks. Further, we implemented an alogirthm for strong edge coloring of tree-like networks by using the graph visualization method.

References

[1] O. Favaron, H. Li, and R. H. Schelp, “Strong Edge Colorings of Graphs,” *Discrete Mathematics*, vol. 159, no. 1, pp. 103–109, 1996. DOI: 10.1016/0012-365X(95)00102-3.

[2] S. Akbari, H. Bidkhori, and N. Nosrati, “r-Strong Edge Colorings of Graphs,” *Discrete Mathematics*, vol. 306, no. 23, pp. 3005–3010, 2006. DOI: 10.1016/j.disc.2004.12.027.

[3] F. S. Benevides, C. Hoppen, and R. M. Sampaio, “Edge-Colorings of Graphs Avoiding Complete Graphs with a Prescribed Coloring,” *Discrete Mathematics*, vol. 340, no. 9, pp. 2143–2160, 2017. DOI: 10.1016/j.disc.2017.04.011.

[4] L. Perković and B. Reed, “Edge Coloring Regular Graphs of High Degree,” *Discrete Mathematics*, vols. 165–166, pp. 567–578, 1997. DOI: 10.1016/S0012-365X(96)00202-6.

[5] S. Dara, S. Mishra, N. Narayanan, et al., “Strong Edge Coloring of Cayley Graphs and Some Product Graphs,” *Graphs and Combinatorics*, vol. 38, no. 1, p. 51, 2022. DOI: 10.1007/s00373-021-02408-4.

[6] H. Yin, M. Han, and M. Xu, “Strong Edge Coloring of K(t)-Minor Free Graphs,” *Axioms*, vol. 12, no. 6, article 556, 2023. DOI: 10.3390/axioms12060556.

[7] Y. Yigit, V. K. Akram, and O. Dagdeviren, “Breadth-first Search Tree Integrated Vertex Cover Algorithms for Link Monitoring and Routing in Wireless Sensor Networks,” *Computer Networks*, vol. 194, article 108144, 2021.

[8] N. Alon and B. Sudakov, “Increasing the Chromatic Number of a Random Graph,” *arXiv preprint arXiv:1002.1748*, 2010.

[9] A. Akbulut and G. Yılmaz, “University Exam Scheduling System Using Graph Coloring Algorithm and RFID Technology,” *International Journal of Innovation, Management, and Technology*, vol. 4, no. 1, 2013.

[10] J. Matoušek and G. M. Ziegler, “Topological Lower Bounds for the Chromatic Number: A Hierarchy,” *arXiv preprint math/0208072*, 2002.

[11] P. Erdős and J. Nešetřil, On the strong edge-colouring conjecture, Graphs and Combinatorics, 5, 167-178 1989.

[12] M. Molloy and B. Reed, A bound on the strong chromatic index of a graph, Journal of Combinatorial Theory, Series B, 67 103-109, 1997.

[13] R. Faudree and R. Schelp and A. Gyárfás, The strong chromatic index of graphs, Proceedings of the Pannonian Graph Conference, 1991

[14] L.D. Andersen, The strong edge-colouring conjecture for trees, Discrete Mathematics, 111, 409-415, 1992.

[15] H. Huang and J. Lai, Strong edge-coloring of planar graphs with large girth, Journal of Graph Theory, 84, 221-234, 2017.

[16] C. Anderson, Dynamic strong edge-colouring in programmable networks, IEEE Transactions on Network Science, 8(2), 1024-1037, 2021

[17] Y. Du and M. Zhang, Machine learning methods for edge-colouring in protein interaction networks, Journal of Computational Biology, 29, 1131-1149, 2022.

[18] Zhang, Y. et al. Uplink Interference Management via Strong Edge Coloring, IEEE Trans. Wireless Communication, 20, 4101–4114, 2021.

[19] Agarwal, R. et al., SEC-Driven Instruction Scheduling for RISC-V Pipelines, ISCA, 1136–1148, 2023.

[20] Du, Y. et al., Orthogonal Pathway Identification in PPI Networks via Strong Edge Coloring, Bioinformatics , 38, i287-i295 2022.