Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 28 No. 5s (2025)

Fluid Flow Numerical Study Considering the Impact of Deborah Number and Slip Condition

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Article History:

Received: *29-10-2024*

Revised: *30-11-2024*

Accepted: *09-12-2024*

Abstract:
This article measures Maxwell fluid flow above a horizontal plate over a porous medium with the influence of MHD. Heat generation and radiation are left for discussion and explanation. The governing equations are converted into nonlinear ODEs by using similarity variables. The nonlinear ODE's answer is obtained using the renowned numerical Runge-Kutta method. The outcome of magnetic field (M), Deborah number (De), porosity (λ), radiation (Rd), Prandtl number (Pr), thermal slip (β), and energy generation (Q) are widely examined over graphs. The visualization and analysis of the consequences of modifying the model's parameters are conducted with the predefined values: Pr = 5, Rd = 2, M = 0.5, Q = 0.3, β = 0.5 and De = 0.6. In order to increase the values of M and Rd, this study attempts to determine the outcomes of thermal slip circumstances and examine the entropy generation and Bejan number. A good agreement is found when the results of the current work are compared with published work for code validation. The research's crucial conclusion is that the M affects both the expansion of thermal fields and momentum. On the other hand, the energy field accelerates due to heat radiation, and the Deborah number is influenced by the magnetic intensity.

Keywords: heat generation, radiation, entropy generation, Maxwell fluid, porous medium, magnetic field.

Nomenclature

Q_0 *Dimensional heat generation*

f *non-dimensional stream Function*

M *Magnetohydrodynamics field*

De *Maxwell fluid parameter (nondimensional)*

Q *nondimensional Heat generation/absorption*

C_p *Specific heat*

Nu *Nusselt number*

q_r *Radiative heat flux*

C_f *Skin Friction Coefficient*

Rd *Radiation parameter*

Ec *Eckert number*

Greek symbol

σ *Electric conductivity*

λ_1 *fluid relaxation time*

ρ *Density*

β *Thermal slip parameter*

υ *Kinematic viscosity*

μ *Dynamic viscosity*

k *Thermal conductivity of the fluid*

λ *nondimensional Porosity parameter*

θ *non-dimensional temperature*

σ^* *Stefan – Boltzmann constant*

ρc_p *Heat capacity*

1. INTRODUCTION

Magnetohydrodynamics is a broad area of study that relates electricity and magnetism and the movement of fluids, particularly plasmas, liquid metals and some electrolytic solutions. In the case of porous flat plates, MHD examines the fluid flow magnetic fields relationship in the presence of a porous medium. The complexity of the study increases even more when considering the subject as a Maxwell fluid, a non-Newtonian fluid with stress-strain rate relationships involving non-integer order fractional derivatives. Maxwell fluids on the other hand have a time related response to a stress and therefore are not Newtonian fluids. The study of the flow of a Maxwell fluid over a porous flat plate requires the assessment of how the magnetic field affects the flow of the fluid and how the absorbent structure affects the flow. Scholars employ these phenomena and develop mathematical modeling first and then solve the governing equations. The resulting equations can contain for example PDEs, describing mass, energy and momentum conservation and the Maxwell fluid constitutive relation. These models give information on the velocity profile, heat transfer, and impact of magnetic field on the Maxwell fluid over the porous-plate plate. Numerically investigating MHD on porous vertical plates with Maxwell fluids gives benefits in different branches of engineering and industries for heat transfer control or for the designing of new materials with special rheological properties. This area of research remains productive and has provided useful information to the two branches; that is, fluid dynamics and electromagnetism.

The non linear approximation to free convection flow through a moving plate with non linear radiation was investigated by Jha and Samaila [1] Recently, based on the same model, Mopuri et al [2] characterized the features of Jeffery fluid flow past an inclined permeable plate. Other related work Mangamma et al [3] examined the effects of radiation and viscous dissipation on a permeable plate with temperature difference by the Galerkin method. The oscillatory precisely investigation of the impact of radiation and chemistry on MHD Cu–water nanofluid flow by using an infinitely oscillatory plate has been done by Saikia et al. [4]. The unsteady radiative MHD flow over a porous stretching plate was studied by Sarma [5]. Saikia and Ahmed [6] studied chemical reaction variable and radiation aspect on unsteady natural convective flow through an infinitely oscillating plate using porous medium. By using machine learning, Reddy et al. [7] studied the inclined plate on the chemically reacting viscoelastic fluid. Ahmad et al [8] synthesized an application of the AFM in scrutinizing thermo diffusion flow of MHD radiative Maxwell fluid confined under slip impact over a surface with energy and mass diffusion. None of the above authors, worked only on the different type of plates with both MHD & Radiation but none working on flat plate only.

In this connection, radiation means the phenomena whereby heat is transported through space by electromagnetic waves without direct contact with suitable materials. Radiation case: when a flat plate is subjected to radiation then it under goes heat transfer process with it surroundings, thus modifying the temperature and altering the convective heat transfer processes. Both mechanisms are crucial to accurately describing the temperature gradients and heat flux within the flow, and thus under standing there interactions is of paramount importance. In many cases heat sources on a flat plate involve internal heating that might include chemical reactions, electrical currents, or other energy losses in the fluid. Heat production being an added parameter in the system adds the complication of the thermal conditions of the system. The analysis of heat generation is especially important in cases where heating is intentional and purposeful, depending on particular manufacturing processes' requirements or electronic devices' designs. Radiation and heat generation on a flat plate are the two areas of concern that are analyzed and modeled mathematically.

Jha and Samaila [9] analyzed the nonlinear approximation for free convection flow across a vertically moving plate with radiation effects. Marinca et al. [10] studied energy transfer in an MHD viscoelastic fluid of the 2 nd grade across a stretching permeable sheet. Iranian et al. [11] discussed the suction injection effect on Maxwell fluid flow on a straight plate with the effect of heat generation. Adnan et al. [12] discovered the impact of energy generation and melting over the MHD flow of Carreau fluid in a permeable medium. Hanif et al. [13] investigated the implication of Cu-Fe_3O_4 on Maxwell fluid flow across a cone. Shah et al. [14] analyzed heat transfer phenomena in MHD flow past a stretchable Riga wall, employing entropy generation rate through a numerical investigation. Rana et al. [15] researched the impact of Hall current and Lorentz force, coupled with radiation, in an inclined slip flow of a couple of stress fluids past a Riga plate. Shankar et al. [16] examined the influences of radiative and viscid dissipative flowing effects on energy and solute transfer in MHD Casson fluid. Nazir et al. [17] investigated the impact of radiative heat flux and heat generation in magnetohydrodynamics natural convection flow of nanofluid inside a porous triangular cavity with thermal boundary conditions. Abideen et al. [18] focused on analyzing second-grade fluid containing carbon nanotubes flowing via an elongated curved surface. Their study considered radiation and internal heat generation effects. In a separate survey, VS et al. [19] discovered the characteristics of magnetohydrodynamics and heat transfer in thermally radiative upper-convected Maxwell fluid flow between moving plates. Ahmad et al. [20] delved into applying the constant proportional Caputo fractional derivative to analyze the thermo diffusion flow of magnetohydrodynamics radiative Maxwell fluid. Their investigation considered slip effects over a flat surface with heat and mass diffusion. Obalalu et al. [21] focused on the energy optimization of quadratic thermal convection in a two-phase boundary layer flow across a moving upright flat plate. Rallabandi [22] investigated finite element solutions for the flow of a non-Newtonian dissipative Casson fluid across a vertically inclined surface, considering constant heat flux, thermal diffusion, and diffusion thermo.

1.1 Application of this study

The analysis of the Maxwell fluid flow on a straight plate with Magnetohydrodynamics, radiation and energy generation, porous media and thermal slip conditions has numerous realistic applications in diverse areas of science and engineering. Here are the potential application. As such, knowledge of the behavior of Maxwell fluids under the effects of MHD, radiation, and thermal influence is important to various industrial applications in material processing industries. Such an understanding can be extended to the enhancement of manufacturing processes in polymer melts, liquid crystals, as well as other fluids that exhibit sophisticated behaviour. Maxwell fluids are employed more and more in reproduction of rheological properties of biological systems. These have implications on biomedical device construction or in drug delivery applications. The study of both Maxwell fluid flow and MHD is associated with flows where the fluid is electrically conducting such as liquid metals. This can be applied in synthesis of new forms of MHD generators, pumps and other devices. The findings of the present work concerning the analysis of Maxwell fluid flow with thermal slip conditions on a flat plate would be of interest in devising micro systems, like heat exchangers, micro channels, etc. Porous media finds wide application in the contemporary engineering sciences such as oil and gas industries, geothermal systems, and fluid filtration. This work can be relevant to enhance the processes taking place in these fields as it illustrates the Maxwell fluid flow in porous media. It turns out that the total losses of heat generation and radiation are large in applications where energy transfer is the major concern. This research may be useful in developing applied heat transfer applications such as solar collectors where radiation is significant. The subject to be discussed is complex fluid flow amended by several parameters of influence, which finds application in studying flows in the environment. That is, such an approach could be useful in studying the methods of pollutants' dispersion in the atmosphere or in water reservoirs. Many applications using renewable energy like solar and wind energy require elements including liquid flow and energy transfer. Thus, this paper aims to investigate the effects of multiple effects on the Maxwell fluid flow and understand how the knowledge gained from it can help in designing and enhancing renewable energy systems.

1.2 Motivation for this study

The rationale for analyzing the Maxwell fluid flow over a straight plate having the features of Magnetohydrodynamics, radiation, porous media, heat generation, and thermal slip are to investigate fluid dynamics and their implications in greater detail. The good understanding of how complex fluids behave when several factors influence them enables enhancing the existing processes in industries. This can enhance both productivity and economy of production processes such as those encountered in materials science involving polymers, liquid crystals or biological fluids. Maxwell fluids when used are commonly employed in the modeling of material with special rheological characteristics. Studying their behavior under different circumstances assists the design of new materials with certain qualities relevant in material science and engineering among other fields. In flows with mhd, thermal, radiation effects, in generators, collectors, hydrodynamic generators, etc the goal is to enhance the conversion processes and efficiencies in energy systems. It could all help to facilitate the construction of more rational, effective and sustainable energy systems. Constructing and analyzing models of fluid behavior are crucial for determining effects on the environment in a large number of situations. For instance, research in fluid dynamics in porous structures helps the analysis of groundwater pollution and pollutant transport which is good for the health of the environment. In biomedical applications, what has been learnt from the Maxwell fluid flow aids in the improvement of the designs of some instruments such as some deliveries of drugs. This can help deliver medications in the human body with a better precision and thus results into quality outcome so desired. Research in the area of multiphase flow with more than one force helps develop technology. The SMEP will provide a research foundation for the design of future technologies as well as enhancement of current technologies for various sectors. All the investigations involved in the study regarding MHD, radiation, heat generation, porous media and thermal slip conditions evident the multidisciplinary nature of the study. This approach enables the researcher to tap from different fields of science with a view of drawing the best from different fields hence coming up with better understanding of the nature of fluid dynamics.

1.3 Novelty of this work

As far as the authors know, no research article has been done to solve Maxwell fluid flow above a flat porous plate with the simultaneous effects of heat generation, magnetic field, and radiation together with porous medium and thermal slip conditions. Hence, the thrust of this research is on MHD over Maxwell fluid over a permeable flat plate involving radiation and energy generation. To solving the issue, Maxwell fluid flow was applied. The heat transport properties of the fluid are evaluated. Arithmetic methods are employed to analyse the model. Although there exist numerous numerical methods which have been described in books and journals, we opted for RK -45 as our approach to computation. Based on all the results interpreted for the profiles and the code validation of this research besides entropy generation and Bejan number analysis as well.

This research addresses the following research queries:

1. How does the incorporation of Magnetohydrodynamics and porous media affect profiles of Maxwell fluid flow over a flat plate?

2. Interactions between heat generation and thermal radiation as well as the Maxwell fluid flow over a flat plate on the overall profile.

3. In what manner do thermal slip conditions influence the profile of an unsteady incompressible Maxwell fluid flow over a horizontal plate?

4. What hend is caused by the combination of MHD and radiation on entropy and Bejan number?

2. SCIENTIFIC MODEL

To build the model, it is assumed that the flow is steady, incompressible, two-dimensional, radiation, energy generation in a porous MHD flat plate. Besides boundary layer calculation the equations which govern the velocity flow and the energy distribution may be included in the familiar representation. The flat permeable plate seen is to a magnetic field of an infinite degree. Here u,v represents mechanisms of velocity in x, y direction respectively associated with uniform stream function $U\infty$,temperature$T\infty$. Here $Tw = T\infty$ the temperature of the wall which is exposed to the fluid flow. We also anticipate that the y-axis is subjected to MHD with magnetic field strength B0 and that the magnetic Reynolds parameter is sufficiently negligible such that the tempted magnetic field can be neglected. Here, T is the temperature while ρ stands for fluid density, υ stands to fluid kinetic viscosity and μ stands for fluid dynamic viscosity. The physical mode of the problem is shown in Figure 1 (Bejan [23]). This system uses Cartesian coordinate with x axis designated for the flow direction and y axis perpendicular to plate flow axis. The following equations are obtained from Mustafa et al. [24].

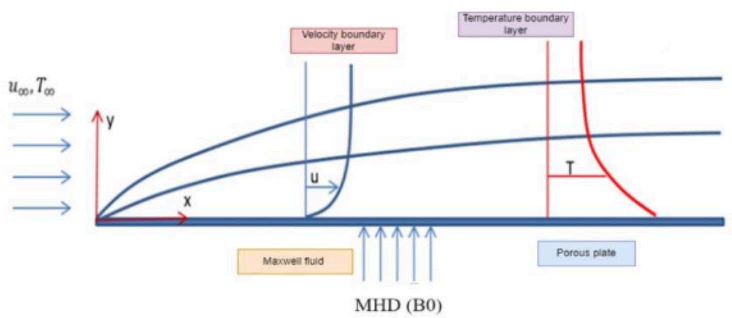

Figure 1. Flow Model

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

The below equation is coupled with the above equation, should be observed as the governing equations for porous and MHD flat plate

Equation of Momentum

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(\frac{\partial^2 u}{\partial y^2}\right)-\frac{\upsilon}{K}u-\frac{\sigma B_0^2}{\rho}u-\lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right) \tag{2}$$

These symbols - k is the kinematic viscosity; σ is the electrically conducting fluid; λ1 is the fluid relaxation time. When radiation, viscous dissipation, and energy generation are added in the energy equation of boundary layer approximations, it may be expressed as below

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+Q_0\left(T-T_\infty\right)+\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Here ρc_p is heat capacity, Q_0 is heat generation/ absorption and Stefan – Boltzmann constant is known as σ^*.

This work is qualitative, and its goals necessitate the assumption of continuous fluid characteristics. To solve the equations mentioned above, boundary conditions are needed, So the borderline limits of the horizontal plate taken from Mustafa et al. [24]

$$at\ \ y=0,\ u=U, v=0,\ -k_f\frac{\partial T}{\partial y}=h_f\left(T_f-T\right)$$

The borderline conditions for the case of a flat plate at a given temperature are

$$at\ \ y\to\infty,\ u=0,\ T=T_\infty \tag{4}$$

Next, nondimensional quantities will be introduced by Mustafa et al. [24] to convert PDEs to ODEs.

$$\eta=\left(\frac{U_\infty}{\upsilon x}\right)^{\frac{1}{2}}y, \psi=\left(\upsilon x U_\infty\right)^{\frac{1}{2}}f, \theta=\frac{T-T_\infty}{T_w-T_\infty}, v=-\frac{\partial\psi}{\partial x}\ \text{and}\ \ u=\frac{\partial\psi}{\partial y} \tag{5}$$

The set of two coupled PDEs can be reduced to the ODEs with the help of similarity variables η and ψ, θ is the dimensionless temperature, f is the dimensionless velocity, U∞ is the velocity stream function, and Tw means the temperature on the wall. Substituting these definitions in the momentum equation we get the following simplified equation.

$$f'''+\frac{1}{2}ff''-\lambda f'-M^2 f'-\frac{De}{4}\left(f^2 f'''+\eta f'^2 f''+f f' f''\right)=0 \tag{6}$$

In this regard, the use of the prime number means freedom from the similarity variables. Here, De symbolizes the material relaxation which is called also the Deborah number, M stands for the magnetic number and λ for porosity.

As we define it, an equation for the thermal boundary layer might be,

$$\left(1+\frac{4}{3}Rd\right)\theta''+\frac{Pr}{2}f\theta'+Pr\,Ec\,f''^{2}+Pr\,Q\,\theta=0 \qquad (7)$$

The Prandtl number Pr, and the energy conservation constant Ec will need to be used in solving this equation. Here Ec is the Eckart number.

With respect to the f, border conditions are as follow are nondimensional as generally written like

$$f=0, f'=1, \theta'=-\beta(1-\theta) \quad at \;\; \eta=0$$

The boundary conditions are in terms of θ (8)

$$f'=0, \theta=0 \;\; at \;\; \eta\to\infty$$

Here β is the thermal slip condition.

Prandtl number $\Pr=\frac{\nu}{\alpha}$, Radiation $Rd=\frac{4\sigma T_{\infty}^{3}}{k\delta}$ and Thermal diffusivity $\alpha=\frac{k}{\left(\rho c_{p}\right)}$.

3. METHOD OF THE SOLUTION AND VALIDATION

This article is focus on the measurement of the Maxwell fluid flow over a straight plate on a porous medium with an influence of magnetic hall diameter. The radiation and the heat production are kept for an explanation for the discussion. By applicability of similarity variables, the governing equations get converted into nonlinear ODE. The present effective numerical RK 45 is utilized to solve the result of the discussed nonlinear ordinary differential equation. The influence of the thermophysical key parameters of M, De, λ, Rd, Pr, β, and Q are described often in graphs. ODE can be solved with the help of a numerical algorithm written in a high-level language on RK-45 method. The selection of mesh results and how to handle the errors relies with the rest of the active results. Hence the fault tolerance of 10-8 and the step size of 0.000001 chosen for the computing technique form good structure for convergence fulfilment. The substitutions for linear ODE are as follows.

$$a_1'=a_2$$
$$a_2''=a_3$$
$$a_3'=-\left[0.5a_1a_3-\frac{De}{4}\left(a_1a_2a_3+\eta a_2a_2a_3\right)-(M^2+\lambda)a_2\right]\left(\frac{1}{1-\frac{De}{4}a_1a_1}\right)$$
$$a_4'=a_5$$
$$a_5'=-\left(\frac{\Pr}{\left(1+\frac{4}{3}Rd\right)}\right)\left[0.5a_1a_5+Eca_3a_3+Qa_4\right]$$

Boundary conditions in linear form

$$a_2 = 1,\ a_1 = 0,\ a_5 = -\beta\left(1 - a_4\right) \quad at \ \ \eta = 0$$
$$a_2 = 0,\ a_4 = 0 \quad at \ \ \eta = \infty$$

Table 1: Results of Skin friction coefficient with the existing reports for distinct values of *De*

De	Irfan et al. [25]	Present Outcomes
0	1.0000000	1.0000001
0.2	1.0518890	1.0518891
0.4	1.1019035	1.1019036
0.6	1.1501374	1.1501375
0.8	1.1967114	1.1967126
1.2	1.2853630	1.2853661

Table 1. Validation table

4. DISCUSSION OF RESULTS

4.1 ENTROPY ANALYSIS

Entropy generation is considered in solving BVPs, particularly in thermal and fluid systems. For the tasks that are irreversible, the principle is used to explain why the system cannot be reversed or undone, For the system to achieve higher efficiency and better performance, this principle is used. It affords a thermodynamic view that gives insight into the physical conditions of the system going on in it. This confirms that a concomitant condition in a system vice is its ability to generate entropy if it is irreversible. In energy transfer or fluid flow problems the irreversibility may be caused by friction, heat conduction and all other dissipative processes. Entropy creation is normally linked with energy dissipation in a certain system. When dealing with entropy generation, engineers and scientists can assess the efficiency of a process. Due to this entropy generation is always the worst possibility which needs to be minimized to enhance the efficiency of energy conversion systems. When entropy generation is included in establishing a BVP, optimization problems that aim at minimizing and controlling entropy may arise. Minimization of the entropy generation can be used to develop systems that are more efficient and of improved environmental status.

The Bejan number and entropy for the given flow problem are given below from [26], [27]

$$S_G = \frac{k}{l^2 T_\infty^2}\left(1 + \frac{16\sigma^* T_\infty^3}{3kk^*}\right)\left(\frac{\partial T}{\partial y}\right)^2 + \frac{\mu}{T_\infty}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\mu}{KT_\infty^2}u^2 + \frac{\sigma B_0^2}{T_\infty^2}u^2$$

By using eqn. (5) entropy generation is a dimensionless form

$$N_G = \mathrm{Re}\left(1+\frac{4}{3}Rd\right)(\theta')^2 + \frac{Br}{\Omega}\left(M^2+\lambda\right)(f')^2 + \mathrm{Re}\frac{Br}{\Omega}(f'')^2$$

$$Be = \frac{\mathrm{Re}\left(1+\frac{4}{3}Rd\right)(\theta')^2}{\mathrm{Re}\left(1+\frac{4}{3}Rd\right)(\theta')^2 + \frac{Br}{\Omega}\left(M^2+\lambda\right)(f')^2 + \mathrm{Re}\frac{Br}{\Omega}(f'')^2}$$

Here Reynolds number $\mathrm{Re} = \frac{U_\infty x}{\upsilon}$

Brinkman number $\left(\frac{\mu U_\infty^2}{k\nabla T}\right)$ is $Br = \frac{\mu}{kT_w - T_\infty}$

The dimensionless temperature difference $\left(\frac{\nabla T}{T_\infty}\right)$ is $\Omega = \frac{T_w - T_\infty}{T_\infty}$

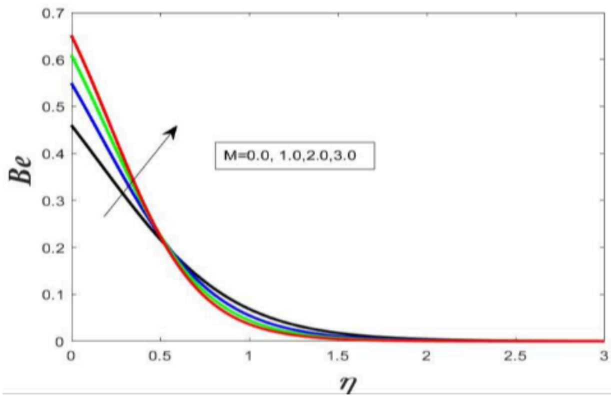

Figure 2. Bejan number profile for the swelling value of *M*

Figure 2 as presented below depicts the Bejan number contour with the increase of M value. Each time M becomes higher, we experience a rise in the profile. In fluid dynamics and energy transfer, the Be refers to a number that defines the ratio of the irreversibility of energy transfer in a system to the total irreversibility present in a system. It is widely employed to explain the origin of the term ‘system’ concerning the consideration of the amount of entropy produced during heat transfer operations. The Bejan number for Maxwell fluid flow on a straight-lined plate with magnetohydrodynamics depends on magnetic impact, flowing fluids, and the condition it sustains. The magnetic field can have a great influence on Bejan numbers in MHD flows. Here are the following possibilities as to why the value of the Be has risen at the profile of Maxwell fluid flow on a flat plate with MHD. Magnetohydrodynamics deals with the interaction of magnetic fields with electrical conducting fluid. Magnetic fields can influence flow mythology and energy exchange phenomena. Augmentation of more magnetic field strength may alter the flow structure and may result in fluctuations of entropy generation and therefore a sound increases of the Be. The MHD effects can produce dissipation of fluid motion and modify the temperature gradients. Coasting of the flow may lead to more irreversibility, which will make the Bejan number to be higher. Such parameters as the flow rate of the fluid and the degree of coupling between the magnetic field and heat exchange surfaces depend on the heat transfer rate. Alterations in the coupling strength may affect the nature of irreversibility dissemination and therefore affect the Bejan number. Occasionally, MHD effects can increase heat transfer coefficients, results that are valuable for a variety of heat transfer applications. This tends to higher irreversible effects of heat transfer which are part of the cause of higher Bejan number. Flows at MHD are complex and cause several flow patterns to occur. The flow structure might prove complex and this might lead to nonuniform distribution of temperature and entropy generation and therefore influence the Be.

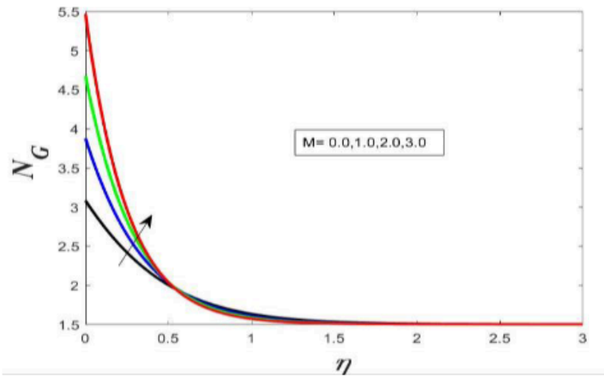

Figure 3. Entropy generation outline for swelling value of *M*

The Entropy generation outline for swelling the value of M is presented in Figure 3 below. Whenever we increase M, the profile becomes higher. The entropy generation profile in the flow of Maxwell fluid past a flat plate under the influence of Magnetohydrodynamics effects depend on several parameters. Possible reasons for the increasing value of entropy generation in such a scenario would be as follows. The effects which can be exhibited by MHD flows are that a generated magnetic field can dampen the flow of the fluid. This damping results into increased irreversibilities of the system, as well as its entropy production. Rising strength of the magnetic field leads to higher damping which in turns increase the entropy generation as described above. It replaces the hydrodynamic behaviour of the fluid with Lorentz forces in magnetohydrodynamics. They can alter or even disrupt the flow pattern, velocity gradients and temperature delivery and hence cause changes in entropy generation. It also revealed that the heat transfer rates in the flow can also be affected by the MHD effects. If due to MHD conditions heat transfer is enhanced, it may lead to the increase in entropy generation. Other losses that occur because of variation in temperature gradients and heat flux patterns elicited by the magnetic field are also possible. In MHD flows there exists a coupling between the magnetic and thermal fields. It was also observed that the distribution of temperature and the change in the magnetic field could influence one another. This coupling can also bring several other sources of irreversibility which will lead to more of entropy production. Such flow structures involve complexities that lead to non-uniformity of temperature and velocity profiles, which are sources of entropy generation.

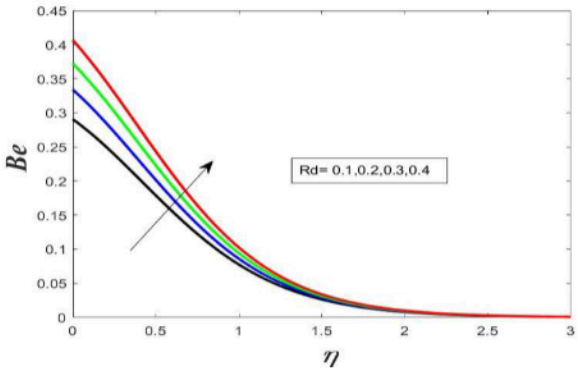

Figure 4. Bejan number profile for the swelling value of *Rd*

In Figure 4, we present the Bejan number profile for swelling the value of Rd. As we can see whenever we grow Rd the profile rises. Th Be is a dimensionless parameter that is employed to describe the degree of irreversibility in the throughput of energy. The aforementioned radiation, sometimes referred to as Rd is used to describe the level and contribution of radiation to total energy transfer. The Be is subjected to alterations in the radiation parameter impacting the design in the following manners. With the increase in the Rd or the ratio of convective heat transfer to thermal radiation, the combined effect of thermal radiation with convection and conduction amongst other modes becomes significant. Radiation brings in extra irreversibility in the fluid flow because it is a non-convective transport process.

As a result, the heat transfer irreversibility increases and thus an increase in the Bejan number. The final mode of energy transfer that is often realized at the high level of the radiation parameter is thermal radiation. They concluded that such dominance may cause variations of the temperature distribution and gradients that characterize entropy generation coupled with energy transport processes. The Bejan number indicates this increased irreversibility owing to the increased dominance of radiation effects. A higher value of the radiation parameter means that, radiation heat transfer cannot be considered insignificantly small in comparison to other modes of heat transfer. The involvement of radiation in the heat transfer increases the complication or irreversible effect which forms the Bejan number. Thermal radiation has influence on temperature distribution on the flat plate. An increase in the dialectical radiation parameter can affect the temperature difference and distribution with respect to the entropy generation which in tern increases the Bejan number.

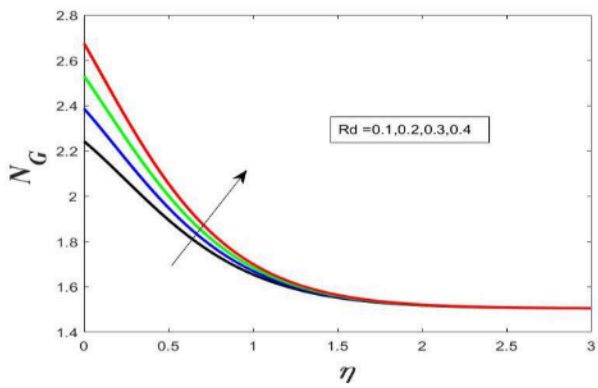

Figure 5. Entropy generation outline for the swelling value of *Rd*

The Entropy generation sketch for increasing the value of Rd is presented in figure 5. Throughout, that raises the profile as soon as we increase Rd. These are possible reasons that caused the entropy generation to increase with the rising of the radiation parameter when the Maxwell fluid flows over a flat plate. A case thus, where the amount of Rd is raised; this means the radiation energy is contributed more to the total energy transfer. In general, it is found that the irreversibility's of radiative heat transfer are greater than those of convective heat transfer. At high frequencies of Rd, the radiation effect becomes prominent; as a result, the entropy generation rate may increase. In the present study, radiation affects the distribution of temperature within the fluid as well as in the vicinity of the flat plate. An increase in the Rd can cause variations in the outlines of temperature and hence can affect temperature gradients. Temperature non-uniformity is also observed on the plates, CFD study shows that the uneven distribution increases the entropy generation. Rd disrupts the heat transfer properties of the fluid in terms of convective heat transfer. These changes in the convective heat transfer caused by the radiative heat transfer effects can reveal new irreversibility's and lead to a higher entropy production. Moreover, potential enhancement in the doses of radiation parameters may result in a rise in the viscosity of the fluids and an increase in viscous dissipation. However, this viscous dissipation is tasked with the responsibility of increasing entropy production of the fluid, and this becomes worse when the fluid is influenced by Maxwell fluid properties which may have complicated rheological characteristics. Thermal radiation modifies boundary layers that, in turn, affect outlines of the velocity and temperature fields close to the flat plate. An increase in radiation affects the boundary layer thickness resulting in high entropy generation in the boundary layer region. This parameter changes the pattern of heat flux distribution over the flat plate. Further entropy generation arises from variations in heat flux principally because of radiative heat transfer processes. The substantial contribution made by radiation in heat transfer increases entropy's productivity. Maxwell fluid properties, which include viscoelastic behavior, work hand in hand with the radiation effect hence causing a blended interaction that increases entropy production. The interaction between radiation and properties of Maxwell fluids may lead to enhanced values of irreversibility.

4.2 Another method of solution

The ODE system with suitable boundary conditions was solved numerically via RK45 with a shooting technique. Graphs are drawn using predetermined values of parameters $\beta = 0.5; Pr = 5; Rd = 2; M = 0.5; Q = 0.3; De = 0.6.$

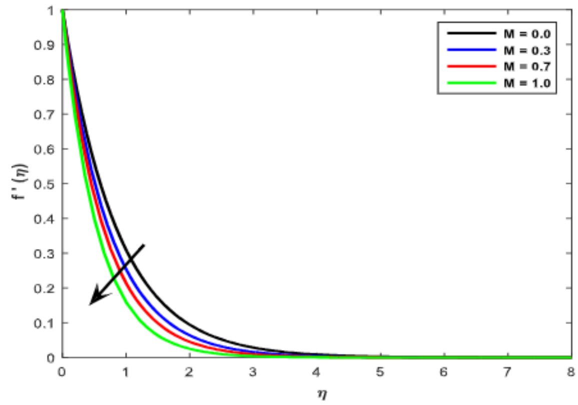

Figure 6. Demonstration of the parameter "M" over the velocity contour.

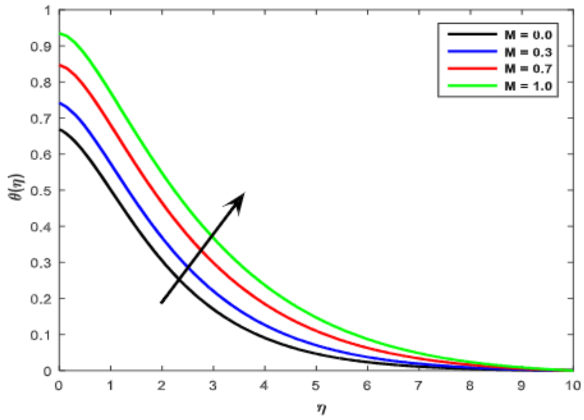

Figure 7. demonstrates the parameter "M" over the temperature contour.

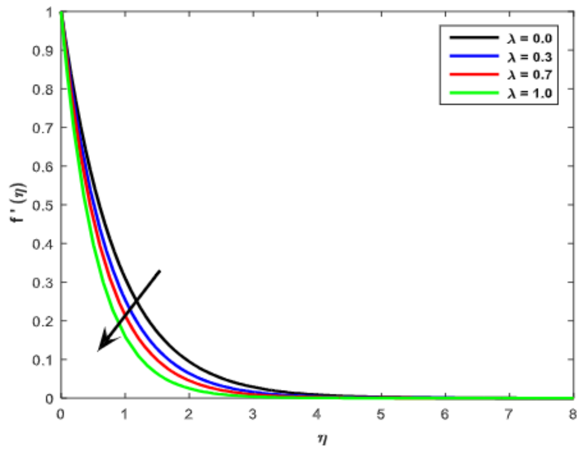

Figure 8. Demonstration of the parameter "λ" over the velocity profile.

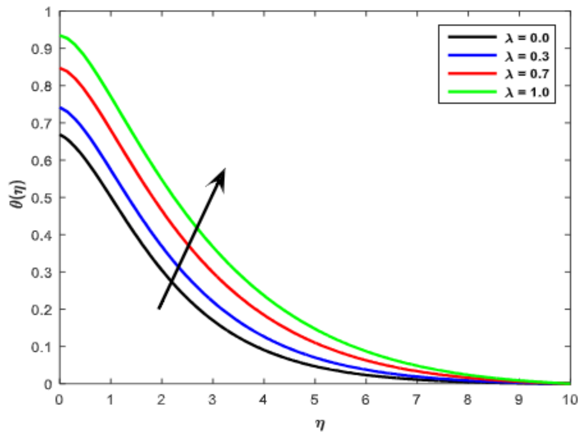

Figure 9. Demonstration of the parameter "λ" over the temperature outline.

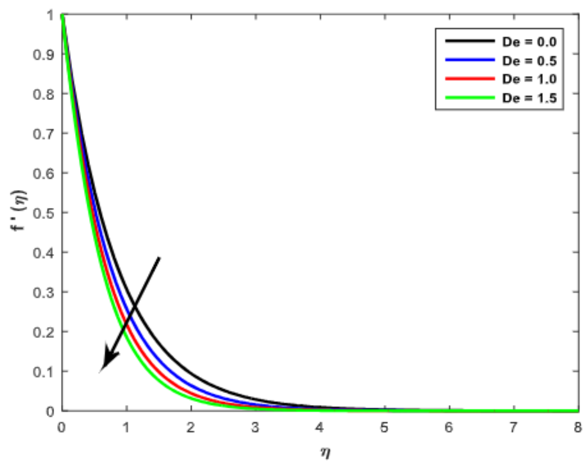

Figure 10. demonstrates the parameter "De" over the velocity profile.

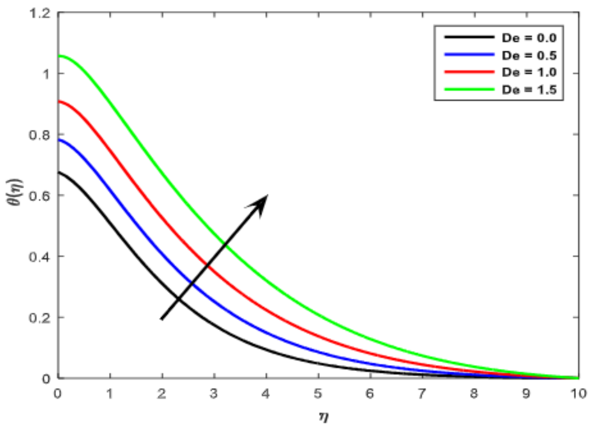

Figure 11. Demonstration of the parameter "De" over the temperature outline.

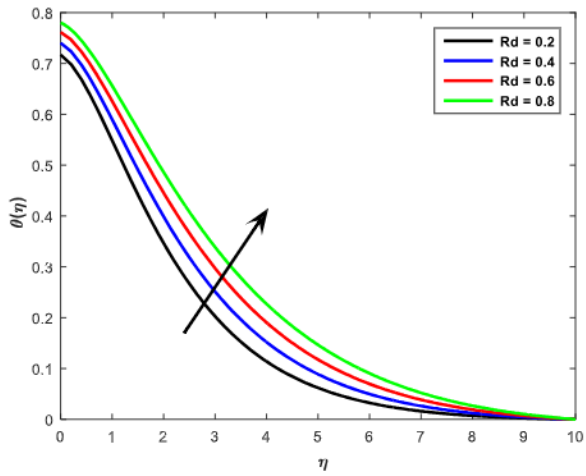

Figure 12. demonstrates the parameter "Rd" over the temperature profile.

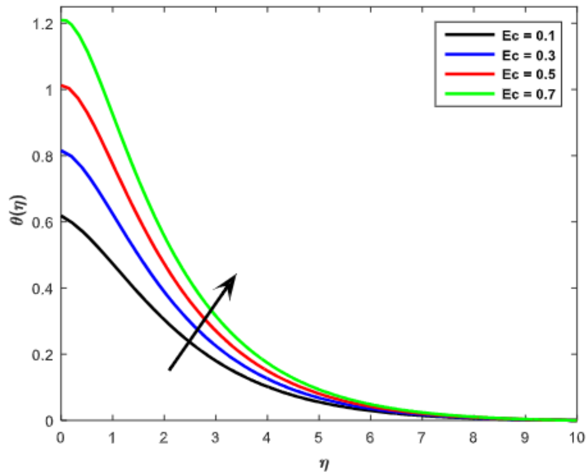

Figure 13. Demonstration of the parameter "Ec" over the temperature profile.

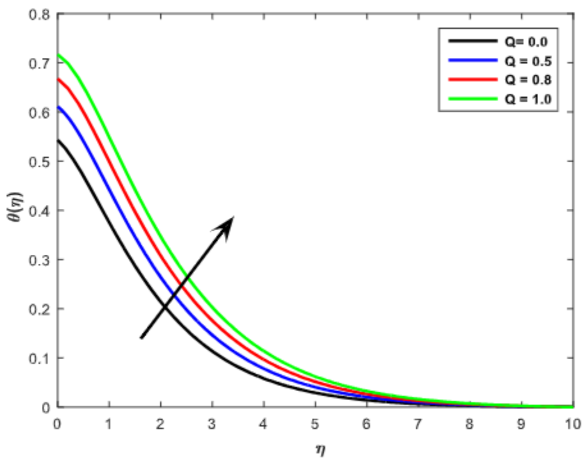

Figure 14. Demonstration of the parameter "Q" on the temperature outline

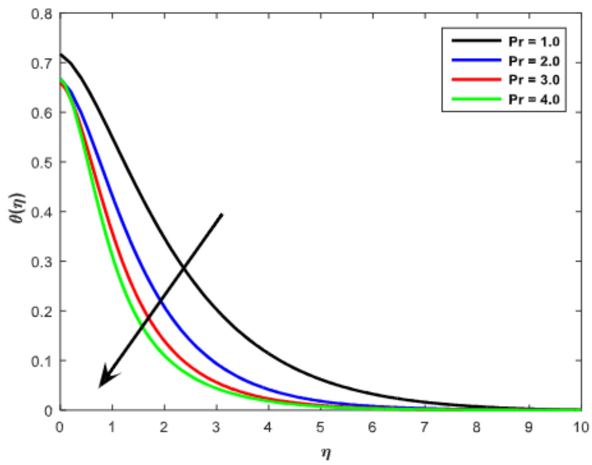

Figure 15. Demonstration of the parameter "Pr" over the temperature outline.

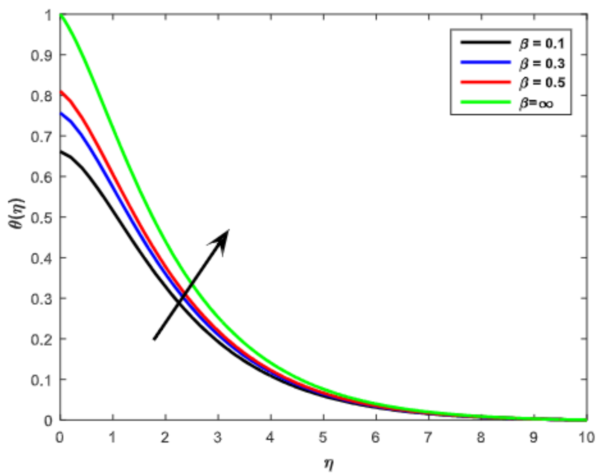

Figure 16. Demonstration of the parameter "β" over the temperature profile.

4.3 Profiles under the Influence of "M"

For the sake of clarity, we also plotted the impression of parameter M on the element velocity and temperature contour in Figure 6-7. Increasing M and the temperature contour augments whilst the velocity profile decreases. It shows a dramatic influence of MHD on the velocity and temperature of the flow field in the fluid flow. MHD is concerned with the effect that a magnetic field when applied to a conducting fluid imposes from changes that it induces in the flow complexities. In its simplest form, MHD still causes changes in the velocity profile of the fluid flow. MHD when applied either perpendicular to the flow direction generates Lorentz forces that counteract the fluid flow. This lowers velocity near the wall, and results in a steeper velocity profile over the boundary layer.

Therefore, the velocity outline becomes horizontal and flow is reduced as magnetic field strength increases. The appearance of the magnetic field defines the impact on the energy transfer and the distribution of temperatures within the frame of the fluid. MHD can disturb the thermal boundary layer and thus change the transfer of energy at the wall. If the magnetic field reduces the velocity of fluid flow near the wall, then in that zone the temperature gradients increase and hence the energy transfer is improved in the particular zone.

4.4 Effect of ‘λ’ over Profiles

The effect of the porosity parameter on velocity and temperature profile is depicted in Figure 8-9. There is an increase in the size of the porosity parameter; growth of the temperature contour is shown, and at the same time a decrease in the size of the velocity contour is observed. The nature of the microstructure determines the porosity of the fluid and the ability of the fluid to drain through the pores. This means that higher porosity is associated with larger interconnectivity of the porous media through which the fluid is to flow thus less flow resistance and consequently higher velocities. However, reduced porosity improves flow resistance and hence the reduction in velocities. This effect is particularly apparent in areas with a higher porosity or void fraction of the nanocomposite material. Porosity variations are also capable of affecting the temperature distribution caused by changes in fluid-solid interaction in porous media. Under these conditions greater porosity enables a greater flow of the fluid and interaction with the surrounding medium, raising the rate of energy transfer while increasing the homogeneity of heat flow. Lower porosity contains the mobility of fluids which may cause a gradient of temperature and relative differences.

4.5 Effect of ‘De’ on Profiles

Figure 10-11 shows the trend of parameter De across the velocity and temperature profile. When De rises, the temperature contours are increased, while the velocity profile is decreased. Here, the relaxation time is of the order of, or greater than the time scale of deformation. The material’s response becomes more flexible and it can memorize previous deformations. The profiles of velocity can be distinguished from Newtonian behaviors and the material might demonstrate elastic response or time delay. Interestingly enough, the actual Deborah number does not affect temperature profiles at all. Nonetheless, the temperature determines the rheological properties of the material and the Deborah number, which defines the material's behavior under deformation. Relaxation time is affected by the changes in the temperature behaving the material sometimes like a fluid and sometimes like a solid. Like temperature, the Deborah number does not alter the basic concentration profiles in which we are interested. Nevertheless, due to the viscoelastic nature of the material, which depends on the Deborah number, the mass transfer of various species in the fluid phase can also be influenced. Surface concentration gradients can influence the relaxation time and behavior of the material, meaning that interactions may be multifaceted.

4.6 Result of 'Rd' on Temperature Outline

Figure 12 below shows the effect of parameter Rd over a temperature profile. All in all, increasing the parameter of Rd highlights the augmentation from a temperature outline. The heat transfer conduction to radiation parameter is called the Rd, which is often denoted as Rd, and is a dimensionless number that quantifies the relation between radiative and conductive energy transfer in a system. In some heat transfer problems where radiation is included electromagnetic waves, for example, infrared radiation are incepted in the heat transfer media. In this regime, the introduced radiative energy transfer is small compared to conductive heat transfer. Here, the principal physical process responsible for the shape of the temperature outline is conduction, and radiation has a very small influence on the temperature. Consequently, the dependences of the local temperature might look like that of an ideally conducting material. As the radiation parameter increases, the value of radiative energy transfer is enhanced as well. The radiative heat transfer is generally more effective in the conduct of heat through larger distances where the media offers low thermal conduction rates. Thus, the temperature profile in situations with a high radiation parameter can differ greatly from a conductively dominated process. The emittance and absorptance of electromagnetic radiation participate in the formation of the thermal gradients. When a medium is heated it produces heat energy through electromagnetic waves. Such waves transfer energy from the heated medium and can be dispersed by other surfaces or media making them hot. When the radiation parameter is small the rate of heat exchange by radiation is less than that by conduction. Such radiation appears to carry the energy from the system off the surface of the material of interest affecting its temperature. Radiative heat transfer vary with the temperature temperature differences, the emissivity of the surfaces as well as the properties of the medium between the two surfaces.

4.7 Effect of 'Ec' over Temperature Contour

The result of the parameter Ec to the temperature contour is shown in Fig 13. Concerning the parameter Ec as it increases, the temperature framework also increases. It is applied in heat transfer studies to determine the interaction between flow and heat transfer processes. The impact of the Ec on the temperature contour is substantial especially where fluid movement is influential in the heat exchange process. In this regime, the variation in the kinetic energy of the flow field is relatively small compared with the variation in the heat energy. The impact of pressure gradients within flow on temperature distribution is not very pronounced and the temperature field may resemble that typical for conductively or convectively dominated systems. This is an indication that the flow of the fluid does have an important impact on heat transfer. In this regime, flow kinetic energy has direct relevance to mixing, heat carriage, and or profile control in the fluid. Convection can move heat by circulating warm or cold fluid in various areas. Mass transfer phenomena involve the change of temperature when fluid parcels with different temperatures are well mixed. Higher Eckert number conditions imply vigorous fluid flow and therefore, convective heat transfer contributes greatly to increasing the temperature gradients. Laminar flow results in the ability of kinetic energy to be lost through viscous processes. This dissipation is then is converted into heat hence contributing to heat transfer. It has also able to show that as the number of Eckert increases the kinetic energy increases the temperature changes. They occur in turbulent flows and result in increased mixing of eddies and vortex plus exchange of energy.

4.8 Outcome of 'Q' on temperature contour

Figure 14 provides an estimation of how parameter Q influenced the temperature structure. As Q increases there is an increment in the temperature contour is evident from the figure above framework. As Q rises, the temperature contour demonstrates an augmentation. The effects of energy generation on the temperature contour were most important in cases, that involve the localized heat sources, influencing the overall temperature field of a system. Conduction is the key mode of heat transfer, they are and the temperature distribution is more influenced by heat flow from high-temperature zones to low-temperature zones. The various heat generation sources may be localized and as a result, may slightly raise the overall temperature in that region, but little affects the global temperature. If heat generation is important compared to other transfer modes, the resulting temperature varies can be very large. Local heat sources increase the quantity of heat supplied to the medium and therefore increase the temperatures close to the bases. These high temperatures can segregate heat away from the original and in the process create a gradient temperature system and more complex temperatures. Heat generation creates heat and in so doing brings into the system more thermal energy. They all said that this energy builds up locally and causes heating close to the source of heat. Dependent on the heat generation rate it gives the rate of energy addition to the system. Whenever heat spreads out from the heat source, then temperature differences start to form. These gradients continue the heat conduction and diffusion thus providing a higher complexity for the temperature delivery than scenarios that do not involve their creation. It is thus important to remark that the nature of the boundary conditions and the geometry of the system influence the temperature field dependence on the heat generation. For instance, if heat generation is at a boundary, the behavior of the boundary condition has the potential to affect the heat transfer or even the temperature distribution.

4.9 Outcome of 'Pr' on temperature outline

It should be noted that, over some parts of the temperature contour, there is a primary effect of the Pr parameter, which is shown in Figure 15. As observed from the graph annotated above, the Pr growths lead to the decline of the temperature contour. with the Prandtl indicating a dimensionless quantity that compares the relative size of momentum diffusivity (kinematic viscosity) and thermal diffusivity in a fluid. It has more importance when convection has a decisive role in heat transfer and is concerned when studying convection. The positive result of the Pr on the temperature profile over a flat plate is significant in the determination of how heat transfer and fluid dynamics affect each other under certain conditions. If the Pr is high as with liquids, the component of thermal diffusivity has a worth more than the component of momentum diffusivity. This mean that the fluid is less capable in transferring momentum and at the same time has the highest ability in transferring heat. In heat transfer on a flat plate, flow visualization suggests that a high Pr leads to a thicker thermal boundary layer and reduced removal rate of heat from the surface. The temperature profile is less oscillatory that suggests slow variation of temperature in the boundary layer.

4.10 Outcome of ‘β’ on temperature contour

As depicted in Figure 16, the reaction of parameter β on the temperature contour can be concluded with an increase in β, the temperature outline also goes up on the temperature contour. As β rises, the temperature outline increases. Thermal slip represents the condition where the temperature of the fluid in contact with the solid wall and that of the wall is different from that of the concrete surface. This situation can occur in flows with nanoscale or microscale volumes of the fluid where molecular actions are prominent. The thermal slip condition raises new issues regarding energy transfer and can alter the temperature distribution above a flat plate. The effect of the thermal slip condition on the temperature distribution across an extensive flat plate is mainly experienced in the case of rarefied gases or where the fluid flow regimes are constrained. In these cases, the walls do not adhere to the traditional slip condition which results from the no-slip temperature boundary conditions.

5. CONCLUSIONS

In this paper, we presented an evaluation of the outcome of the Maxwell fluid flow in the presence of radiative magnetohydrodynamics with a flat plate embedded with the fluid. Besides boundary conditions, heat transfer and viscous dissipation were assumed for this work. Velocity and temperature solutions in terms of numbers are obtained by solving the nonlinear governing equations with the developed RK45 method. The data is plotted graphically, and it is discovered that these parameters play a crucial role in the flow field and a few other physical variables of interest. The graphs were obtained with predetermined values of the parameters. The analysis of the results is given in terms of graphical interrelations, and the conclusion is made that these parameters greatly affect the rate of flow and some other physical characteristics. Plots were made with fixed parameters as indicated earlier. $Pr = 5$; $Rd = 2$; $M = 0.5$; $Q = 0.3$; $\beta = 0.5$; $De = 0.6$. This study seeks to establish the thermal slip conditions and, in the process, come up with entropy analysis and the Bejan number while tracing the results for varying values of M and Rd.

The resulting are

1. It is observed that the entropy generation profile increases with the increase in values of M and Rd.

2. As for Bejan’s number profile, M and Rd in its denominator showed that more of it was enhanced with the rise in the value of M and Rd.

3. As the value of M, lambda and De increased, the momentum boundary layer was seen to reduce in value.

4. The increase of Pr influences a decrease of the energy boundary layer flow viscosity while the energy boundary layer flow increases with De, Ec, Q, Rd, M, β.

5. The magnetic intensity plays a role on Maximum parameter.

5.1 Future work

This manuscript has the potential for expansion by including additional elements such as the mass equation, introduction of various parameters, modification of the solution methodology, adjustment of boundary conditions, and exploration of different geometries.

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