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Development of optimal reduced-order model for gas turbine power plants using particle swarm optimization technique

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Summary

Analysis of higher-order gas turbine plant in real time would be tedious and expensive. In order to overcome this complexity, reduced-order model for 5001M heavy-duty gas turbine rated 18.2 MW has been obtained by Routh approximation, clustering technique, modified pole clustering, eigen permutation, Mihailov criterion, and Padé approximation algorithms. The step responses are obtained using MATLAB/Simulink and compared based on time domain specifications and performance index criteria. It indicates that the mixed method, namely, Routh approximation–Padé approximation algorithm–based reduced-order model, retains the original characteristics. Further, particle swarm optimization (PSO) algorithm has also been applied to develop an optimal reduced-order model. Based on the dynamic response against the load disturbance and set point variations, PSO-based reduced-order model has been identified as an optimal reduced-order model for heavy-duty gas turbine. The reduced-order model proposed in this paper will be suitable for analyzing the dynamic behavior of heavy-duty gas turbine plants in real-time environment.

KEYWORDS

heavy-duty gas turbine, model order reduction technique, optimal reduced-order model, particle swarm optimization, simplified model, speedtronic governor

1 | INTRODUCTION

Utilization of renewable energy sources such as solar, wind, and biomass for power generation has become a viable solution to reduce greenhouse gas emission.[1] India, being the bioenergy-rich country with 25-GW potential, has completed 500 biomass power and cogeneration projects by the end of 2017.[2] Heavy-duty gas turbine (HDGT) has attracted the power industry because of numerous advantages such as better energy conversion efficiency and fuel flexibility.[3] HDGT has been used for power generation by either combined cycle or simple cycle operation mode. Even though the HDGT in combined cycle operation could increase the thermal efficiency to 60%, gas turbines with simple cycle operation have more operational flexibility, less start-up time, and load following capacity.[4,5] Speedtronic governor control system for gas turbine has evolved from the electronic type control up to the digital implementation for an effective control and operation.[6,7] Droop governor mode has been found as a suitable governor option for grid-connected operation.[8] Rowen developed a mathematical model for the dynamic simulation of the HDGT plants in simple cycle operation.[3] The dynamic behavior of 150-MW HDGT was analyzed using this model, and the responses were validated.[9] Careful modeling and control of HDGT plants are required to avoid inevitable shutdown in grid connected operation.[10,11] In order to improve the dynamic and steady-state response of higher-order HDGT plants, attempts were made to develop proportional-integral-derivative (PID) controller and soft computing techniques–based controllers such as neural network, fuzzy logic, and neuro-fuzzy technique.[12-15]

Panda et al revealed that the design, analysis, and controller synthesis for any higher-order system in real-time environment are tedious and expensive.[16] In order to overcome this complexity and reduce the computational burden, it is very much essential to identify an equivalent reduced-order model, which would retain the dominant characteristics of the original system.[17] It is understood from the literature review that only few attempts were made to identify the reduced-order model of the real-time system, viz, wind turbine blade system and micro-turbine system.[18,19] Though the previous researchers used the Rowen (1983) model only for dynamic analysis and controller development for HDGT[3,8,10,12-,15,20] there were no attempts made to identify the reduced-order model of the HDGT plant. For the dynamic studies of HDGT with less computation, it is essential to develop the reduced-order model. Therefore, the authors have proposed in this paper to develop the reduced-order model of speedtronic governor–controlled HDGT developed by General Electric Co, USA. For achieving the reduced-order model for the HDGT, an equivalent higher-order transfer function need to be identified. The authors developed the higher-order transfer function of 5001M HDGT model using superposition principle and validated its dynamic behavior with original system.[21]

In this paper, it is proposed to identify an optimal reduced-order model for typical HDGT by using the equivalent higher-order model named linear transfer function model (LTFM) as presented in Iqbal et al.[21] Section 2 briefs about the simplified model of HDGT as proposed by Rowen.[3] Model order reduction techniques, namely, Routh approximation,[22,23] clustering technique,[24,25] modified pole clustering,[26-28] eigen permutation algorithm,[29] and Mihailov criterion[30,31] along with Padé approximation,[24,32,33] mainly deal with the stability of large-scale system. Therefore, these methods are used for obtaining the reduced-order model of large-scale HDGT as presented in Section 3. The simulation results have been compared with the original HDGT plant, based on time domain specifications and various performance index criteria. Further, it is proposed to identify an optimal reduced-order model by optimizing the reduced-order model coefficients using PSO algorithm as given in Section 4. In order to ensure the stability of HDGT, the step response of reduced-order model of HDGT have been analyzed against the load disturbance and set point variations as explained in Section 5.

2 | SIMPLIFIED MODEL OF GAS TURBINE PLANT

The overall transfer function model of the HDGT developed by Rowen[3] includes various limiters, fuel system, compressor, and turbine system. Modeling of speed governor, acceleration, and temperature controllers have also been presented in Iqbal et al[13] and Balamurugan et al.[15] Low value select (LVS) block in the dynamic simulation model selects the limiter whose output is minimum corresponding to minimum fuel requirement. The fuel system dynamics of the gas turbine consists of valve positioner and fuel system actuator. The fractional amount as about 23 percentage of the rated fuel is required to support self-sustaining of the gas turbine under no-load condition. The gas turbine dynamics include the torque function F2.

The acceleration controller of the gas turbine is useful only during start-up time, and it will be inactive, when the frequency deviation is not greater than ±1 percentage. The temperature controller takes the control action only when the exhaust temperature exceeds the limit. Based on this behavior, the acceleration and temperature controllers were eliminated, and the simplified model was identified as shown in Figure 1.[15] As the speedtronic governor droop setting varies between 2 to 10 percentages, the authors had optimized the droop setting by genetic algorithm. The optimal droop setting for all the HDGT plants is found to be around 4 percentages.[20]

Hannett and Afzal Khan (1993) presented the model parameters of typical gas turbine model and case study–based models.[34] Further, the typical model of HDGT was found to be more optimistic as presented in Hannett and Khan.[34] Therefore, the typical model of HDGT was used for dynamic studies.[8,10,12-15] It is also understood from the LTFM given in Rezaei et al[18] that the HDGT plants are higher order in nature. The higher-order system is fairly complex, which often makes it difficult to understand the original behavior of the system in real-time implementation. Controller development for the higher-order system and analyzing its response in real time would

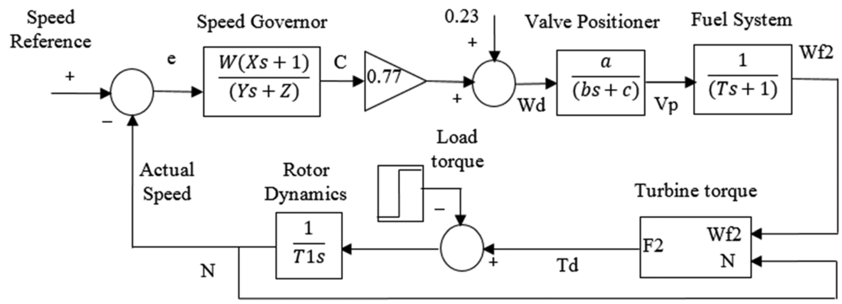

FIGURE 1 Simplified transfer function model of HDGT plant

be difficult and expensive.[15] In order to reduce the hardware complexity, it is necessary to obtain the reduced-order model of the HDGT plant without affecting its original behavior. Authors had verified an effectiveness of some of the reduction techniques using case study–based gas turbine plant.[35] Therefore, it is proposed to identify the reduced-order model of a typical HDGT plant by model order reduction techniques as explained in Section 3.

3 | MODEL ORDER REDUCTION OF HDGT PLANTS

Reduced-order model of any higher-order system can reduce the hardware complexity as well as the computation burden.[17] As the dynamic behavior of all HDGT models proposed by Rowen is found to be almost similar,[13] model order reduction procedure for 5001M HDGT plant has been presented in this section. The model parameters of 5001M HDGT have been furnished in Appendix A. Initially, an equivalent higher-order transfer function of 5001M HDGT was obtained by authors, and it has been expressed in Equation (1).[21] The higher-order (nth-order) transfer function of HDGT is considered to be in the form as given in Equation (2) with d_0, d_1, … … , d_{n-1} as numerator coefficients and c_0, c_1, … … , c_n as denominator coefficients. This paper proposes to identify an equivalent reduced-order (rth order) transfer function of HDGT with t_0, t_1, … … , t_{r-1} as numerator coefficients and u_0, u_1, … … , u_r as denominator coefficients in the form as given in Equation (3).

$$G(s) = \frac{-0.04932\,s^3 - 2.0953\,s^2 - 23.734\,s + 1514.13}{s^4 + 42.53s^3 + 501.3s^2 + 1015s + 1576}, \tag{1}$$

$$\mathrm{G(s)} = \frac{\mathrm{P(S)}}{\mathrm{Q(S)}} = \frac{d_0 + d_1 s + d_2 s^2 + \ldots d_{n-1} s^{n-1}}{c_0 + c_1 s + c_2 s^2 + \ldots c_n s^n}, \tag{2}$$

$$\mathrm{Gr(s)} = \frac{\mathrm{P_r(S)}}{\mathrm{Q_r(S)}} = \frac{t_0 + t_1 s + t_2 s^2 + \ldots t_{r-1} s^{r-1}}{u_0 + u_1 s + u_2 s^2 + \ldots u_r s^r}. \tag{3}$$

In a grid connected system, small disturbances can cause sustained oscillation that may lead to system outage.[15,17] Therefore, it is necessary to analyze the performance of HDGT using model order reduction methods that uses stability criterion.[27] As the stability of the reduced-order model is influenced by the numerator of the original system,[36,37] the mixed approach has been preferred for model order reduction. Padé approximation is a traditional method of rational fraction approximation and uses the coefficients of Taylor series expansion of a function f(x).[38,39] Padé approximation will fetch the solution even if the Taylor series does not converge. This method is computationally simple and may lead to superior performance when the function contains the poles.[38] Because of these reasons, Padé approximation (PA) technique has been widely used for model order reduction.[24] In this paper, the denominator polynomial of the reduced-order model is obtained by Routh approximation (RA), clustering technique (CT), modified pole clustering (MPC), eigen permutation (EP), and Mihailov criterion (MC) techniques, and the numerator polynomial is derived using Padé approximation (PA) as explained below.

3.1.1 | Routh approximation–Padé approximation (RA-PA) algorithm

In this section, the reduced-order model of 5001M HDGT is obtained by combining the Routh approximation (RA) along with Padé approximation (PA) algorithm. All the sequences of approximants in Routh approximation technique converge monotonically to the original higher-order system. The approximants will be stable, if the original higher-order system is stable.[22,23] RA-PA algorithm for the model order reduction is furnished below.

Step 1: By using the denominator coefficients (c_0,c_1,c_2. ...) of the higher-order LTFM of 5001M model as referred in Equation (1), the alpha Routh parameters α_1 and α_2 are obtained and presented in Table 1.[22]

By substituting the "α" parameters in the recursive equation shown in Equation (4), the denominator polynomial, $Q_r(s)$, of the reduced-order model is obtained as in Equation (5).

$$Q_r(s) = 1 + \alpha_2 s, + \alpha_1 \alpha_2 s^2 \tag{4}$$

$$Q_r(s) = s^2 + u_1 s + u_0. \tag{5}$$

Step 2: Based on the coefficients of denominator polynomial, the numerator polynomial of the reduced-order model, $P_r(s)$ is identified by using Padé approximation (PA) algorithm.[24] The power series expansion of the higher-order HDGT is expressed by Equation (6).

$$G(s) = e_0 + e_1 s + e_2 s^2 + \ldots\ldots \tag{6}$$

where

$$e_0 = \frac{d_0}{c_0}; e_1 = \frac{1}{c_0}[d_1 - c_1 e_0]$$

From the coefficients of power series expansion (e_0 and e_1) and the reduced-order denominator coefficients (u_0 and u_1) obtained using RA method, the numerator polynomial of the second-order 5001M HDGT, $P_r(s)$, is obtained as shown in Equation (7).

$$P_r(s) = t_0 + t_1 s, \tag{7}$$

where $t_0 = u_0 e_0$.

$$\mathrm{t}_1 = \mathrm{u}_0 \mathrm{e}_1 + \mathrm{u}_1 \mathrm{e}_0.$$

TABLE 1 Alpha Routh table

$c_0^0 = c_0$ $c_0^1 = c_1$	$c_2^0 = c_2$ $c_2^1 = c_3$	$c_4^0 = c_4$
$\alpha_1 = \frac{c_0^0}{c_0^1}$	$c_0^2 = c_2^0 - \alpha_1 c_2^1$	–
$\alpha_2 = \frac{c_0^1}{c_0^2}$	–	–

Step 3: Then the reduced-order model, $G_r(s)$, of the 5001M HDGT plant is obtained by combining the denominator and numerator polynomials, $Q_r(s)$ and $P_r(s)$, respectively, and the step response is analyzed as given in Section 5.

3.1.2. | Clustering technique–Padé approximation (CT-PA) algorithm

Sinha and Pal (1990) proposed the model order reduction procedure by forming the clusters of both the poles and zeros.[25] The criterion for pole clustering in clustering technique (CT) is based on the relative distance between the higher-order poles and the order of the reduced-order model of HDGT plants.[24] Then the cluster centers are formed using inverse distance measure criterion. Initially, the pole clusters of the original higher-order HDGT plants are generated, based on the nature of the poles whether real or complex conjugate poles. In this section, clustering technique–Padé approximation (CT-PA) combination is used to obtain the reduced-order model of 5001M HDGT plant. Clustering technique has been used to derive the denominator polynomial, and the numerator polynomial is obtained by Padé approximation algorithm as given below.

Step 1: The denominator polynomial of the reduced-order model is identified using clustering technique as detailed below. Initially, the poles of the higher-order HDGT shown in Equation (1) are obtained and presented in Equation (8).

$$\text{Poles} = [-20.2235 \pm 2.06071i] \text{and} [-1.0415 \pm 1.6520i]. \tag{8}$$

By considering these complex conjugate poles as in the form $(\varphi_1 \pm j\theta_1)$ and $(\varphi_2 \pm j\theta_2)$, the real and imaginary parts of complex pole cluster center are obtained by inverse distance measure method using Equations (9) and (10), respectively. The complex pole cluster center has been expressed in the form $(C_c \pm jD_c)$.

$$C_c = \left[\left(\sum_{i=1}^{2}\left(\frac{1}{\varphi_i}\right)\right)/2\right]^{-1}, \tag{9}$$

$$D_c = \left[\left(\sum_{j=1}^{2}\left(\frac{1}{\theta_j}\right)\right)/2\right]^{-1}. \tag{10}$$

By substituting the complex pole cluster center in Equation (11), the denominator polynomial, $Q_r(s)$, is obtained as shown in Equation (12).

$$Q_r(s) = [s-(C_c + jD_c)] \times [s-(C_c - jD_c)], \tag{11}$$

$$Q_r(s) = s^2 + u_1 s + u_0. \tag{12}$$

Step 2: By using the coefficients of power series expansion, e_0 and e_1 along with u_0 and u_1, the reduced-order numerator polynomial is obtained by Padé approximation (PA) as given in Section . The reduced-order numerator polynomial is expressed as given in Equation (13).

$$P_r(s) = t_1 s + t_0. \tag{13}$$

Step 3: The reduced-order model of 5001M HDGT plant using CT-PA algorithm is obtained by combining the denominator and numerator polynomial of the reduced-order model, $Q_r(s)$ and $P_r(s)$, respectively. Then the step response of the reduced-order model is analyzed as given in Section 5.

3.1.3. | Modified pole clustering–Padé approximation (MPC-PA) algorithm

The reduced-order denominator polynomial obtained by CT may sometimes become unstable, though the original higher-order system is stable.[28] In order to overcome this drawback, effective pole cluster centers are generated by modified pole clustering (MPC) technique.[27,28] In this section, modified pole clustering (MPC) technique is used to obtain the reduced-order denominator polynomial of 5001M HDGT plant. Further, the Padé approximation (PA) algorithm is used to obtain the numerator polynomial. The reduced-order model is obtained using MP-PA algorithm in two steps as detailed below.

Step 1: Considering that the complex conjugate poles presented in Equation (8) are in the form, $(\varphi_1 \pm j\theta_1)$ and $(\varphi_2 \pm j\theta_2)$, the denominator polynomial is obtained by using MPC algorithm based on the modified pole cluster center of the form, $(C_m \pm jD_m)$. By substituting the real pole cluster center obtained using Equation (10) and the real part of first complex conjugate pole, φ_1 in Equation (14), the real part of modified pole cluster center, C_m, is obtained.

$$C_m = \left[\left(\frac{-1}{|\varphi_1|} + \frac{-1}{|C_c|}\right)/2\right]^{-1}. \tag{14}$$

Similarly, the imaginary part of modified pole cluster center, D_m, is obtained by using the imaginary pole cluster center obtained from Equation (9) and the imaginary part of complex conjugate pole, θ_1. It is expressed in Equation (15). The complex form of modified pole cluster center is presented in the form $(C_m \pm jD_m)$.

$$D_m = \left[\left(\frac{-1}{|\theta_1|} + \frac{-1}{|D_c|}\right)/2\right]^{-1}. \tag{15}$$

Then the denominator polynomial is obtained by substituting the modified pole cluster center in Equation (16). It is expressed by $Q_r(s)$ as in Equation (17).

$$Q_r(s) = [s - (C_m + jD_m)] \times [s - (C_m - jD_m)], \tag{16}$$

$$Q_r(s) = s^2 + u_1 s + u_0. \tag{17}$$

Step 2: The reduced-order numerator polynomial is obtained by Padé approximation algorithm by using power series expansion coefficients (e_0 and e_1) and the coefficients of denominator polynomial (u_0 and u_1) as discussed in Section . It is denoted by $P_r(s)$ as given in Equation (18).

$$P_r(s) = t_1 s + t_0. \tag{18}$$

Step 3: By using the denominator and numerator polynomials, $Q_r(s)$ and $P_r(s)$, respectively, the reduced-order model of 5001M HDGT plant using MP-PA algorithm is obtained, and the step response is obtained as explained in Section 5.

3.1.4. | Eigen permutation–Padé approximation (EP-PA) algorithm

Eigen permutation (EP) algorithm is one of the stability criterion–based reduction technique. This is an efficient, computer-oriented and simple algorithm.[29] Eigen permutation (EP) algorithm is used in this work to obtain the denominator polynomial of the reduced-order model of HDGT plants, and the numerator polynomial of the reduced-order model is obtained by using Padé approximation (PA) algorithm. The EP-PA algorithm for model order reduction is presented below.

Step 1: Initially, the reduced-order denominator polynomial of 5001M HDGT is obtained using the eigen permutation (EP) algorithm as shown in Figure 2.[29]

Let λ_1 and λ_2 be the complex conjugate poles of the higher-order gas turbine model in the form as referred in Equations (19) and (20), respectively.

$$\lambda_1 = (Re.\varphi_1 \pm Im.\theta_1), \tag{19}$$

$$\lambda_2 = (Re.\varphi_2 \pm Im.\theta_2). \tag{20}$$

Since the values of $Re.\ \varphi_1$and $Re.\ \varphi_2$ are not same, the values of $Re.\ \lambda_{e1}$and $Im.\ \delta_{e1}$ are computed from Equations (21) and (22), respectively.

$$\text{Re}.\lambda_{e1} = \frac{1}{Re.\varphi_2}\sum\nolimits_{j=1}^{2} Re.\varphi_j, \tag{21}$$

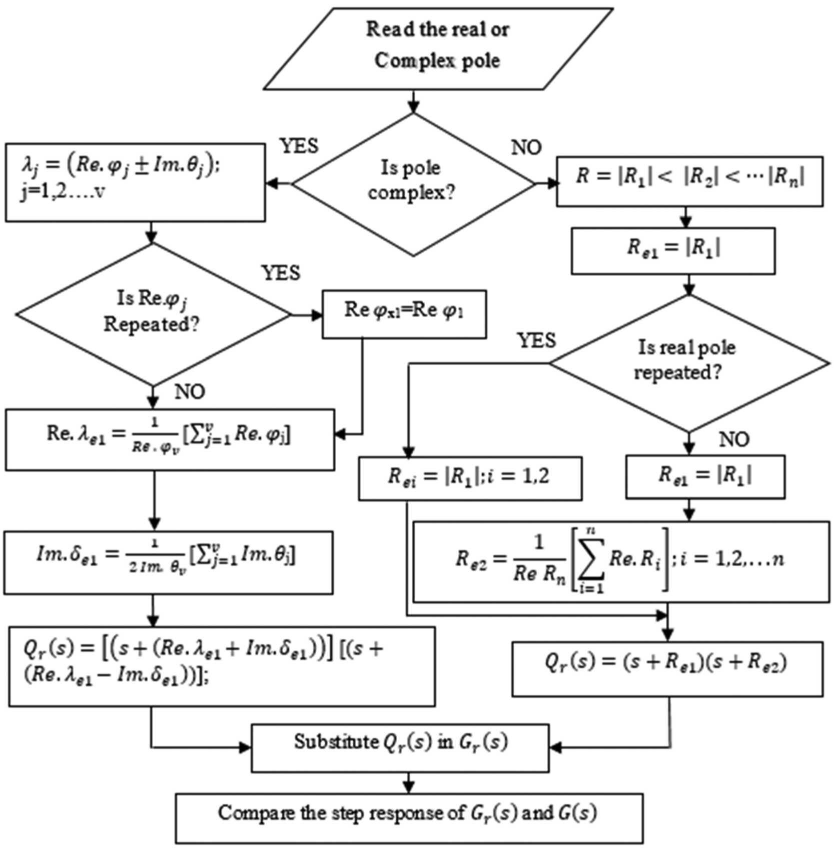

FIGURE 2 Flow chart for eigen permutation algorithm

$$\text{Im}.\delta_{e1} = \frac{1}{2.Im.\theta_2}\sum_{j=1}^{2} Im.\theta_j. \tag{22}$$

On substituting the values of Re. λ_{e1} and Im. δ_{e1} in Equation (23), the reduced-order denominator polynomial, $Q_r(s)$, is obtained as expressed in Equation (24).

$$Q_r(s) = [(s + (Re.\lambda_{e1} + Im.\delta_{e1}))][(s + (Re.\lambda_{e1} - Im.\delta_{e1}))], \tag{23}$$

$$\mathbf{Q_r(s) = s^2 + \mathit{u}_1\,\mathit{s} + \mathit{u}_0}. \tag{24}$$

Step 2: Then the reduced-order numerator polynomial of 5001M HDGT plant is obtained using the Padé approximation (PA) algorithm. Based on the denominator coefficients (u_0 and u_1) obtained using EP algorithm and the coefficients of power series expansion (e_0,e_1), the numerator polynomial, $P_r(s)$ is obtained as shown in Equation (25).

$$P_r(s) = t_1\,s + t_0. \tag{25}$$

Step 3: The reduced-order model of 5001M HDGT plant by eigen permutation–Padé approximation (EP-PA) algorithm is obtained by using the denominator and numerator polynomials as given in Equations (24) and (25), respectively. Then the step response of the reduced-order model is analyzed as presented in Section 5.

3.1.5. | Mihailov criterion–Padé approximation (MC-PA) algorithm

Mihailov criterion (MC) technique is another stability-based reduction technique.[30] In this algorithm, Mihailov criterion (MC) is used to obtain the denominator polynomial of the reduced-order model and the Padé approximation (PA) algorithm to obtain the numerator polynomial involving the following steps.

Step 1: The denominator polynomial of the reduced-order model is obtained using Mihailov criterion method. Let the denominator polynomial of the higher-order HDGT plant be represented as shown in Equation (26). On Substituting, $s = j\omega$, Equation (26) can be rewritten as shown in Equation (27).

$$\mathbf{Q(s) = 1576 + 1015\,s + 501.3\,s^2 + 42.53\,s^3 + s^4}, \tag{26}$$

$$\mathbf{Q(j\omega) = 1576 + j1015\,\omega - 501.3\,\omega^2 - j42.53\,\omega^3 + \omega^4}. \tag{27}$$

In order to obtain the intersecting frequency of the higher-order system, the real and imaginary parts of Equation (27) are separated and equated to zero as represented in Equations (28) and (29), respectively.

$$\mathbf{1576 - 501.3\,\omega^2 + \omega^4 = 0}, \tag{28}$$

$$\mathbf{1015\,\omega - 42.53\,\omega^3 = 0}. \tag{29}$$

The intersecting frequencies, ω_0,ω_1,ω_2, and ω_3 are to be arranged such that $\omega_0 < \omega_1 < \omega_2 < \omega_3$. It gives $\omega_0 = 0$, $\omega_1 = 1.7787$, $\omega_2 = 4.8852$, and $\omega_3 = 22.319$. For any reduced-order model to be stable, the Mihailov frequency characteristics must alternatively interact with abscissa and ordinate same as that of the higher-order system.[30] Since the number of intersections would be equal to the order of characteristics polynomial, first two intersecting frequencies

ω_0 and ω_1 are considered for the model order reduction. From these values, the denominator polynomial of the reduced-order model, $Q_r(j\omega)$, in complex conjugate form is expressed as in Equation (30).

$$Q_r(j\omega) = \sigma_1\left(\omega^2 - \omega_1^2\right) + \sigma_2(j\omega). \tag{30}$$

The values of σ_1 and σ_2 are obtained by equating the real and imaginary parts of Equation (30) with Equations (28) and (29), respectively. While equating the real and imaginary parts, the value of "ω" is assumed to be "ω_0" and "ω_1," respectively. Then the value of σ_1 and σ_2 along with ω_1 is substituted in Equation (30), and the denominator polynomial of the reduced-order model, $Q_r(j\omega)$, is obtained as shown in Equation (31).

$$Q_r(j\omega) = -498.14\,\omega^2 + 1576 + j\,880.445\,\omega. \tag{31}$$

Then by replacing $j\omega = s$ in Equation (31), the reduced-order denominator polynomial, $Q_r(s)$, of 5001M HDGT plant is obtained as expressed in Equation (32).

$$\mathbf{Q_r(s) = s^2 + \boldsymbol{u_1}\,s + \boldsymbol{u_0}}. \tag{32}$$

Step 2: The numerator polynomial of the reduced-order model is obtained using Padé approximation (PA). Based on the reduced-order denominator polynomials (u_0,u_1) and the coefficients of power series expansion (e_0,e_1), the numerator polynomial is obtained as shown in Equation (33).

$$P_r(s) = t_1\,s + t_0. \tag{33}$$

Step 3: The reduced-order model of 5001M HDGT plant, $G_r(s)$, is obtained by combining $Q_r(s)$ and $P_r(s)$ using Mihailov criterion–Padé approximation (MC-PA) algorithm.

The simulation results of all the reduced-order models obtained by various reduction techniques namely RA-PA, CT-PA, MPC-PA, EP-PA, and MC-PA are compared with that of the original higher-order HDGT. On comparing the simulation results, the effective reduced-order model is identified and the results are furnished in Section 5.

4 | PSO ALGORITHM FOR OPTIMAL REDUCED-ORDER MODEL

In order to obtain the optimal reduced-order model for the HDGT plants, it is attempted in this paper to optimize the coefficients of the effective reduced-order model by using evolutionary technique. Particle swarm optimization (PSO) algorithm has gained numerous advantages such as simple concept, easy implementation, and better computational efficiency.[40-42] PSO and genetic algorithms are the most promising algorithms comparing with all the heuristic technique.[16] PSO algorithm is found easy to implement as it does not use the concept of mutation or crossover like genetic algorithm.[43] Hence, the PSO algorithm has been used in this paper for identifying the optimal reduced-order model of HDGT plants.

In PSO algorithm, each single solution in search space moves with an adoptable velocity and position to reach the best solution. In such a search space, every particle has its best fitness value called "Pbest," and the overall best fitness value of all the particles is called as "Gbest".[44] In this paper, the optimal reduced-order model is identified using PSO algorithm by developing MATLAB coding as given below.

Step 1: Initially, the reduced-order and higher-order transfer functions are fed as the input to the PSO algorithm.
Step 2: The weight parameter, w, acceleration factors denoted by, C_1 and C_2, and the random numbers, namely, $rand_1$ and $rand_2$, are initialized.
Step 3: Number of particles and number of generations are initialized.

Step 4: The coefficients of the effective reduced-order model are optimized by updating the velocity and position of the particles using PSO algorithm so as to match the response of the reduced-order model with that of the original higher-order system. The velocity and position at ($k+1$)th iteration are updated based on the velocity and position of the particle at kth iteration as shown in Equations (34) and (35), respectively.

$$V_i^{k+1} = w^* V_i^k + \left[C_1{}^* rand_1{}^* \left(Pbest_i{}^k - S_i{}^k\right)\right] + \left[C_2{}^* rand_2{}^* \left(Gbest_i{}^k - S_i{}^k\right)\right] \tag{34}$$

$$S_i^{k+1} = S_i^k + V_i^{k+1}. \tag{35}$$

Step 5: Minimization of sum of squared error between the unit step response of SM, g(t), and the unit step response of the effective reduced-order model, r(t), as shown in Equation (36) has been used as a fitness function.

$$\mathrm{P_{SSE}} = \sum [\mathrm{g(t)} - \mathrm{r(t)}]^2\, \mathrm{dt}. \tag{36}$$

Step 6: By running the MATLAB code for minimizing the fitness function, the numerator and denominator coefficients of the reduced-order model of 5001M HDGT are optimized.
Step 7: Optimization is stopped, if the stopping criteria is satisfied.

The coefficients of the effective reduced-order model have been optimized by this algorithm using MATLAB. Then the step responses are compared based on time domain specifications and performance indices as presented in Section 5.

5 | SIMULATION RESULTS AND DISCUSSION

In this section, the reduced-order models of the 5001M HDGT are obtained by the model order reduction techniques as explained in Section 3. In this section, the dynamic behavior of these reduced-order models has been analyzed against the load disturbance and set point variations. Then the optimal reduced-order model has been identified using PSO algorithm. The reduced-order model obtained by RA-PA, CT-PA, MPC-PA, EP-PA, and MC-PA methods is expressed in Equations (37) to (41), respectively.

$$\mathbf{G_r(s)} = \frac{\mathbf{P_r(s)}}{\mathbf{Q_r(s)}} = \frac{\mathbf{-0.0551\,s + 3.4793}}{\mathbf{s^2 + 2.332\,s + 3.6216}}, \tag{37}$$

$$\mathbf{G_r(s)} = \frac{\mathbf{P_r(s)}}{\mathbf{Q_r(s)}} = \frac{-0.8122\,s + 7.0}{s^2 + 3.962\,s + 7.287}, \tag{38}$$

$$\mathbf{G_r(s)} = \frac{\mathbf{P_r(s)}}{\mathbf{Q_r(s)}} = \frac{\mathbf{-3.7062\,s + 16.1272}}{\mathbf{s^2 + 7.217\,s + 16.787}}, \tag{39}$$

$$\mathbf{G_r(s)} = \frac{\mathbf{P_r(s)}}{\mathbf{Q_r(s)}} = \frac{\mathbf{-225.7895}\,s + \mathbf{401.712}}{s^2 + \mathbf{40.8354}\,s + \mathbf{418.1451}}, \tag{40}$$

$$\mathrm{G_r(s)} = \frac{\mathbf{P_r(s)}}{\mathbf{Q_r(s)}} = \frac{-0.3072\,s + 3.0394}{s^2 + 1.7674\,s + 3.1637}. \tag{41}$$

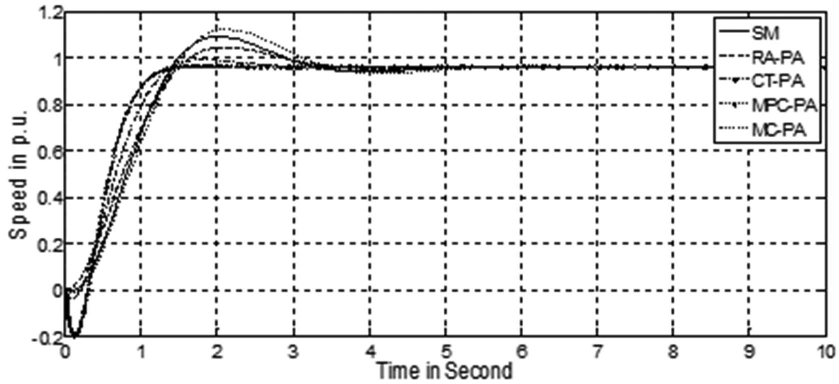

FIGURE 3 Step responses using various reduced-order models

TABLE 2 Time domain specifications using reduced-order models

Model	Time Domain Specifications Mp, pu	Tr, s	Ess, pu
SM	0.092	0.914	0.039
RA-PA	0.045	1.0585	0.039
CT-PA	0.0	0.8763	0.039
MPC-PA	0.00	0.7092	0.039
MC-PA	0.1221	0.9494	0.039

Then the reduced-order models are implemented in MATLAB/Simulink and analyzed their dynamic performances against the load disturbances. A unit step load of 1 pu magnitude is applied at 1 second with the total simulation period of 10 seconds. The step response of 5001M HDGT plant using the reduced-order models is obtained and compared with that of higher-order model named simplified model (SM).

Since the step response of EP-PA based reduced-order model deviates very much from the desired response, the response of RA-PA, CT-PA, MPC-PA, and MC-PA based reduced-order models is only compared with that of simplified model (SM) as shown in Figure 3. Time domain specifications such as maximum peak overshoot (Mp), rise time (Tr), and steady-state error (Ess) have been obtained as shown in Table 2.

The results show that the peak overshoot, rise time, and steady-state error using RA-PA based reduced-order model is closer to that of SM. In order to validate the results further, the error between the simplified model response, g(t), and the reduced-order model response, r(t), is obtained for all the reduced-order models and analyzed their dynamic performance using error criteria. It is obvious that lesser the error criteria value, much closer is the response of reduced-order model with the original system. Equations (42), (43), and (44) show various error criteria, namely, integral of squared error (ISE), integral of time multiplied with the squared error (ITSE), and integral of absolute error (IAE). Table 3 shows the error criteria of various reduced-order models of 5001M HDGT plant.

$$P_{ISE} = \int [g(t) - r(t)]^2 \, dt, \tag{42}$$

$$P_{ITSE} = \int [g(t) - r(t)]^2 \, t \, dt, \tag{43}$$

$$P_{IAE} = \int |(g(t) - r(t))| \, dt. \tag{44}$$

TABLE 3 Error criteria using reduced-order models

Model	Error Criteria		
	P_{ISE}	P_{ITSE}	P_{IAE}
RA-PA	0.003025	0.00611	0.111
CT-PA	0.02164	0.03084	0.2499
MPC-PA	0.05614	0.05997	0.3808
MC-PA	0.004766	0.006247	0.1212

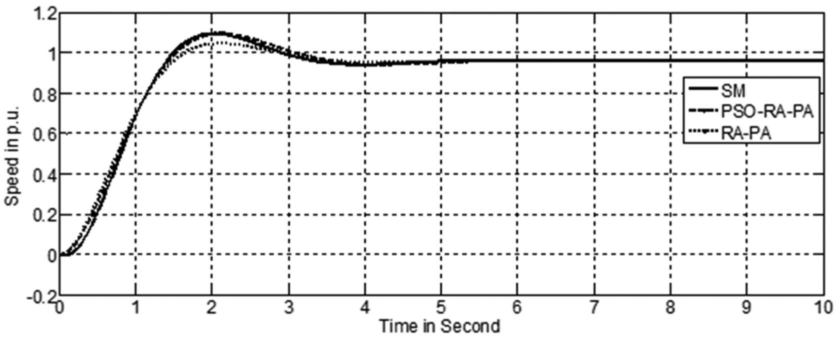

FIGURE 4 Comparison of reduced-order model responses of 5001M

It is witnessed that all the error criteria obtained using RA-PA based reduced-order model are found to be lesser than that of other reduced-order models. The step response of RA-PA based reduced-order model is also identified as close to the original system response. Thus, the time domain specifications and the error criteria ensure that the RA-PA algorithm-based reduced-order model retains the original characteristics. Therefore, the RA-PA based reduced-order model is identified as an effective reduced-order model for the 5001M HDGT plant in grid-connected operation.

For improving the response of the reduced-order model further, it is attempted to optimize the coefficients of RA-PA based reduced-order model using PSO algorithm as explained in Section 4. The MATLAB code is made to run up to 100 generations with 80 particles, and the numerator and denominator coefficients of the RA-PA based reduced-order model are optimized using PSO algorithm. At the end of 100 generations, the close match between the higher-order SM and the PSO-based reduced-order model is achieved with the fitness value of 0.0161. The reduced-order model optimized using PSO algorithm is denoted by PSO-RA-PA based reduced-order model and expressed as $G_{r,o}(s)$ as shown in Equation (45).

$$\mathbf{G_{r,o}(s)} = \frac{\mathbf{-0.05454s + 3.0333}}{\mathbf{s^2 + 1.8953s + 3.1570}}. \tag{45}$$

Further, the step response of the PSO-RA-PA based reduced-order model has been obtained and compared with that of the RA-PA based reduced-order model and the higher-order SM. A unit step load is applied at 1 second, and step responses are obtained by simulating the MATLAB/Simulink models up to 10 seconds. Figure 4 shows the step response of 5001M HDGT model with PSO-RA-PA based reduced-order model, RA-PA based reduced-order model, and SM. The respective time domain specifications are obtained and compared in Table 4.

It is identified that the peak overshoot, rise time, and steady-state error using PSO-RA-PA based reduced-order model are closer to that of SM than the RA-PA based reduced-order model. Then the error between the unit step response of higher-order simplified model and the PSO-RA-PA based reduced-order model are obtained. Based on the error between the simplified model response, g(t), and the PSO-RA-PA based reduced-order model response, r(t), various error criteria as expressed in Equations (42), (43), and (44) are obtained. Table 5 shows the error criteria of PSO-RA-PA based reduced-order model and RA-PA based reduced-order model.

The error criteria, P_{ISE}, P_{ITSE}, and P_{IAE}, using PSO-RA-PA based reduced-order model are very much less than the RA-PA based reduced-order model. It is an indication that the PSO-RA-PA based reduced-order model yields the response very much closer to the simplified model response. It also witnesses that the PSO-RA-PA based reduced-order model retains the original characteristics of the HDGT power plants than RA-PA based reduced-order model.

TABLE 4 Comparison of time domain specifications of 5001M

Model	Time Domain Specifications Mp, pu	Tr, s	Ess, pu
SM	0.092	0.914	0.039
PSO-RA-PA	0.0937	1.0144	0.039
RA-PA	0.045	1.0585	0.039

TABLE 5 Comparison of error criteria of 5001M model

Model	Error Criteria P_{ISE}	P_{ITSE}	P_{IAE}
PSO-RA-PA	0.001612	0.002074	0.07319
RA-PA	0.003025	0.00611	0.111

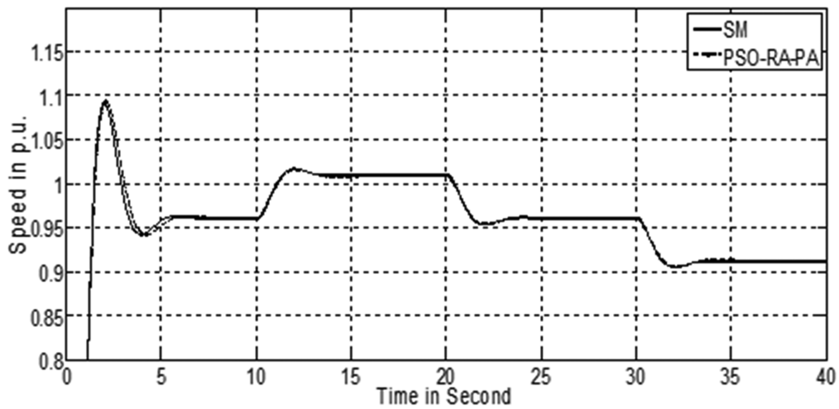

FIGURE 5 Step response of PSO-RA-PA and SM for ±0.05 pu set point variation

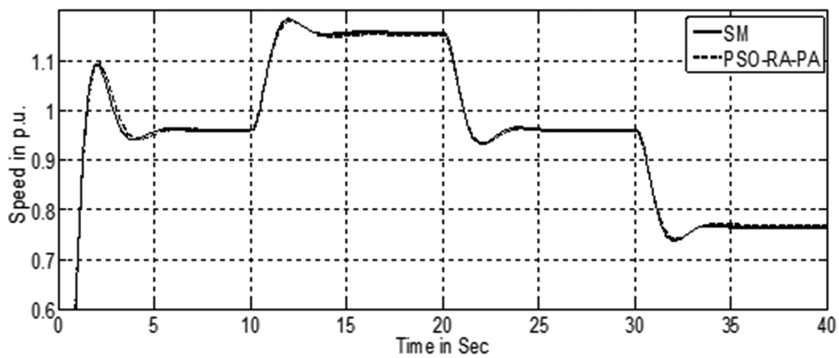

FIGURE 6 Step response of PSO-RA-PA and SM for ±0.2 pu set point variation

In grid-connected HDGT power plants, set point variation is one of the major concerns. Therefore, the effectiveness of the PSO-RA-PA based reduced-order model of 5001M HDGT has also been demonstrated against the set point variations. Since the speed limit of HDGT is between 95 to 105 percentages,[3] the step response of 5001M HDGT using PSO-RA-PA based reduced-order model and the higher-order SM against ±0.05 pu set point variations. The step response of 5001M plant using the PSO-RA-PA based reduced-order model is obtained against four set points, namely, 1.0, 1.05, 1.0, and 0.95 pu, which are set at 0, 10, 20, and 30 seconds, respectively, with the total simulation period of 40 seconds. Then the simulation results are compared with that of the simplified model response as shown in Figure 5. Similarly, the dynamic performance of the models has also been tested against the set point that is far from the nominal set point value. Figure 6 shows the dynamic behavior of simplified model and PSO-RA-PA based reduced-order model for ±0.2 pu set point variation.

The simulation results against the set point variations also witnesses that the PSO-RA-PA based reduced-order model tracks the set point variation as fast as that of the SM of the 5001M HDGT power plant. Based on all these analysis, the PSO-RA-PA based reduced-order model is identified as an optimal reduced-order model for the **HDGT power plants.**

6 | CONCLUSION

In this paper, the optimal reduced-order model is identified for all HDGT plants in grid-connected operation. Various model order reduction techniques, namely, RA, CT, MPC, EP, and MC, have been used to obtain the denominator polynomial, and the well-known PA algorithm is used to obtain the numerator polynomial of the reduced-order model. The effectiveness of these reduced-order models are demonstrated using time domain specifications and performance index criteria. The simulation results indicate that RA-PA based reduced-order model retains the original characteristics than other model order reduction techniques. Then the coefficients of RA-PA based reduced-order model are optimized using PSO algorithm and found that the dynamic and steady-state response are improved by the PSO-RA-PA based reduced-order model. Hence, the PSO-RA-PA based reduced-order model is identified as an optimal reduced-order model for all HDGT plants in grid connected operation. The optimal reduced-order model of HDGT plants identified in this paper can further be used to develop the controller and analyze the behavior of HDGT plants in real-time environment.

LIST OF SYMBOLS AND ABBREVIATIONS

W, X, Y, Z	Speed Governor coefficients; W=1/Droop; Droop=4%; X=0; Y=0.05; Z=1
a, b, c	Valve positioner Transfer function Coefficients; a=1; b=0.05; c=1
Wd	Fuel demand signal in per unit
Wf2	Fuel flow in per unit
N	Turbine rotor speed in per unit
Td	Turbine torque in per unit; Td=F2= 1.3 * (Wf2-0.23) + 0.5 * (1-N)
Vp	Valve positioner signal in per unit
T	Fuel system time constant; T=0.4
T1	Rotor time constant; T1=16.2
G(s)	Transfer function model of higher order HDGT
$G_r(s)$	Transfer function model of reduced order HDGT
λ_j	Complex conjugate poles of higher order gas turbine model
$\varphi_1, \varphi_2 \ldots, \varphi_v$	Real part of complex pole of original system
$\theta_1, \theta_2 \ldots \theta_v$	Imaginary part of complex pole of original system
$R_1, R_2, \ldots . R_n$	Real poles of original higher order system
C_c, D_c	Real and imaginary parts of complex conjugate pole cluster centres
C_m, D_m	Real and imaginary parts of modified pole cluster centres
$d_0, d_1, \ldots .$	Co-efficient of higher order numerator polynomial
$c_0, c_1, \ldots .$	Co-efficient of higher order denominator polynomial
$t_0, t_1, \ldots .$	Co-efficient of reduced order numerator polynomial
$u_0, u_1, \ldots .$	Co-efficient of reduced order denominator polynomial
ω	Angular frequency, rad/sec
$\omega_0, \omega_1, \ldots . \omega_3$	Intersecting frequencies of higher order system
σ_1, σ_2	Constant used in denominator reduced order polynomial
V_i^k, V_i^{k+1}	Velocity of i^{th} individual at k^{th} and $(k+1)^{th}$ iteration
w	Weight parameter
C_1, C_2	Acceleration factors; $C_1=0.5$, $C_2=0.5$
$rand_1, rand_2$	Random numbers between 0 and 1; $rand_1 = 1$, $rand_2 = 0.5$
S_i^k, S_i^{k+1}	Position of i^{th} individual at k^{th} and $(k+1)^{th}$ iteration
$P_{best,i}^k$	Best position of i^{th} individual at k^{th} iteration
$G_{best,i}^k$	Best position of the i^{th} group at k^{th} iteration

g(t)	Unit step response of Simplified model
r(t)	Unit step response of effective reduced order model
PSO	Particle Swarm Optimization
HDGT	Heavy Duty Gas Turbine
PID	Proportional-Integral-Derivative
LTFM	Linearized Transfer Function Model
SM	Simplified Model
LVS	Low Value Select
PA	Pade Approximation
RA	Routh Approximation
CT	Clustering Technique
MPC	Modified Pole Clustering
EP	Eigen Permutation
MC	Mihailov Criterion
ISE	Integral of Squared Error
ITSE	Integral of Time multiplied with Squared Error
IAE	Integral of Absolute Error
RA-PA	Routh Approximation-Pade Approximation
CT-PA	Clustering Technique-Pade Approximation
MPC-PA	Modified Pole Clustering-Pade Approximation
EP-PA	Eigen Permutation-Pade Approximation
MC-PA	Mihailov Criterion-Pade Approximation
PSO-RA-PA	Particle Swarm Optimization-Routh Approximation-Pade Approximation

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APPENDIX A

W, X, Y, Z	Speed governor coefficients; W = 1/Droop; Droop = 4%; X = 0; Y = 0.05; Z = 1
a, b, c	Valve positioner transfer function coefficients; a = 1; b = 0.05; c = 1
Wd	Fuel demand signal in per unit
Wf2	Fuel flow in per unit
N	Turbine rotor speed in per unit
Td	Turbine torque in per unit; Td = F2 = 1.3 * (Wf2-0.23) + 0.5 * (1-N)
Vp	Valve positioner signal in per unit
T	Fuel system time constant; T = 0.4
T1	Rotor time constant; T1 = 16.2
G(s)	Transfer function model of higher-order HDGT
$G_r(s)$	Transfer function model of reduced-order HDGT
λ_j	Complex conjugate poles of higher-order gas turbine model
$\varphi_1, \varphi_2 \dots, \varphi_v$	Real part of complex pole of original system
$\theta_1, \theta_2 \dots \theta_v$	Imaginary part of complex pole of original system
$R_1, R_2, \dots .R_n$	Real poles of original higher-order system
C_c, D_c	Real and imaginary parts of complex conjugate pole cluster centers
C_m, D_m	Real and imaginary parts of modified pole cluster centers
$d_0, d_1, \dots .$	Co-efficient of higher-order numerator polynomial
$c_0, c_1, \dots .$	Co-efficient of higher-order denominator polynomial
$t_0, t_1, \dots .$	Co-efficient of reduced-order numerator polynomial
$u_0, u_1, \dots .$	Co-efficient of reduced-order denominator polynomial
ω	Angular frequency, rad/s
$\omega_0, \omega_1, \dots .\omega_3$	Intersecting frequencies of higher-order system
σ_1, σ_2	Constant used in denominator reduced-order polynomial
V_i^k, V_i^{k+1}	Velocity of ith individual at kth and $(k+1)$th iteration
w	Weight parameter
C_1, C_2	Acceleration factors; $C_1 = 0.5$, $C_2 = 0.5$
$rand_1, rand_2$	Random numbers between 0 and 1; $rand_1 = 1$, $rand_2 = 0.5$
S_i^k, S_i^{k+1}	Position of ith individual at kth and $(k+1)$th iteration
$P_{best,i}^k$	Best position of ith individual at kth iteration
$G_{best,i}^k$	Best position of the ith group at kth iteration
g(t)	Unit step response of Simplified model
r(t)	Unit step response of effective reduced-order model